Probing ultracold gases at short distances

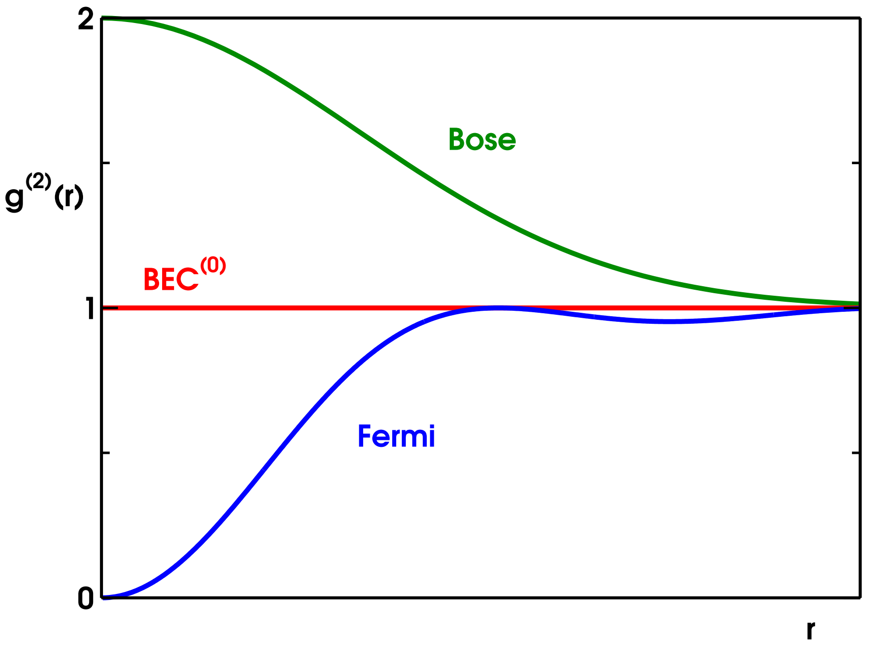
Collège de France, October 20th, 2021

- **Hanbury Brown and Twiss correlations in a BEC**

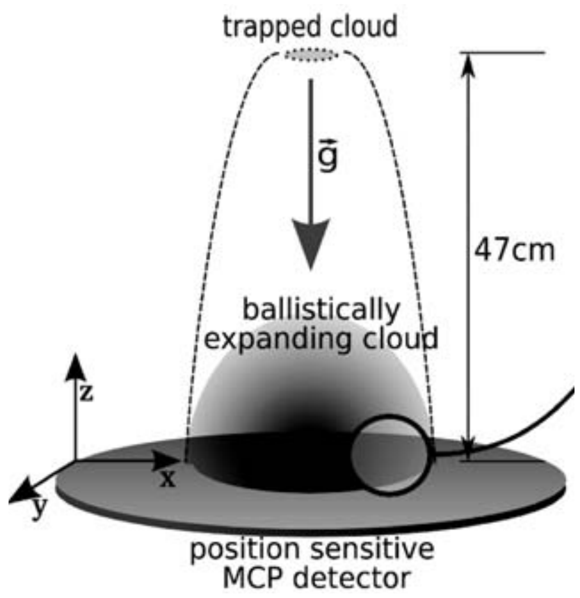
$$\langle \hat{n}(\mathbf{x})\, \hat{n}(0) \rangle = n\,\delta(\mathbf{x}) + n^2 g^{(2)}(\mathbf{x})$$

bunching of bosons $g_{\mathrm{id}}^{(2)}(0) = 2$

disappears in a BEC Kagan et al 1985

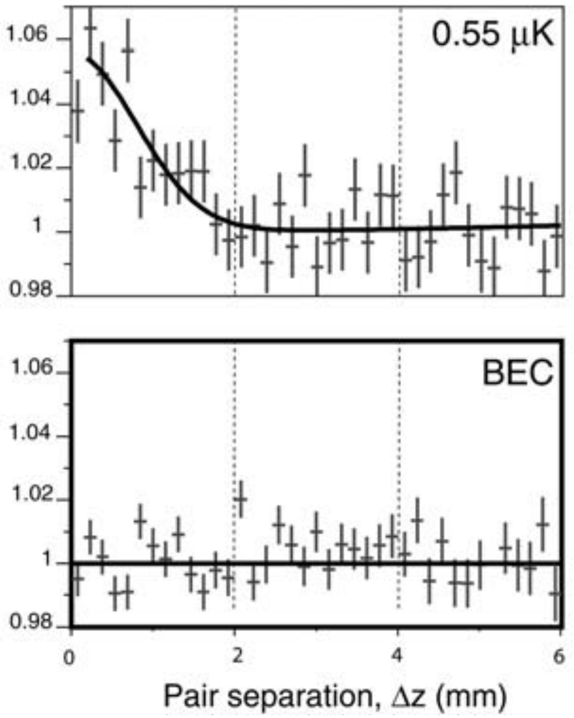


measured by single-atom detection with metastable He-atoms



Schellekens et al 2005

Öttl et al 2005



• suppression of bunching by repulsive interactions

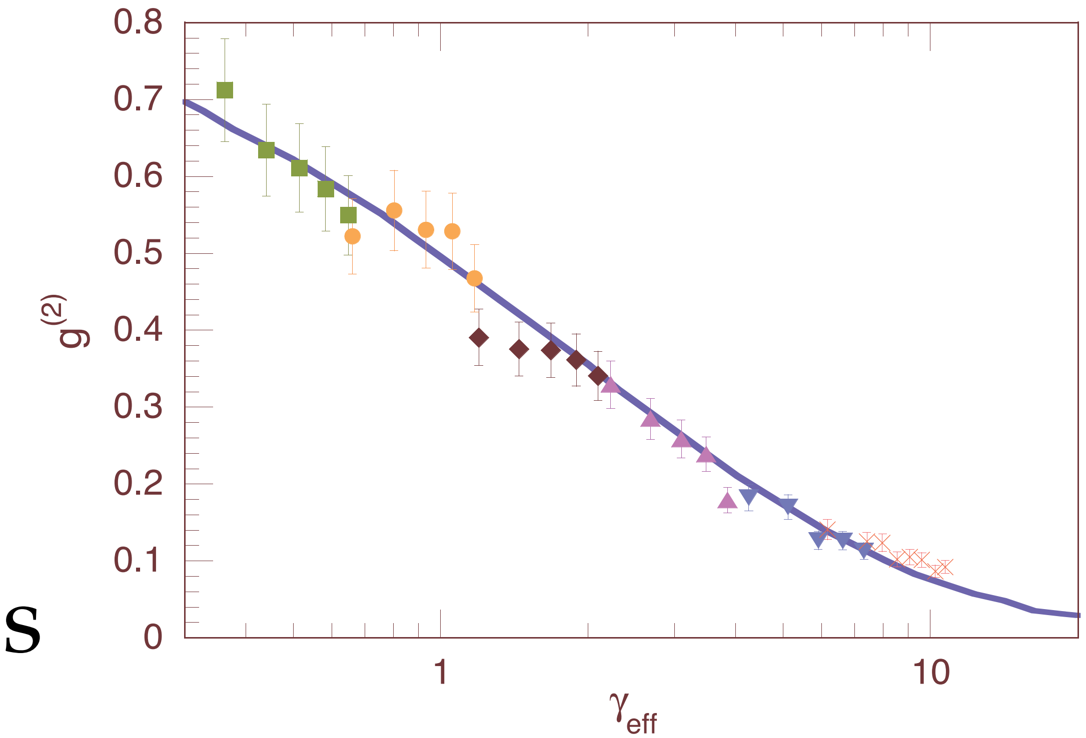
zero-range interactions in one dimension $V(x) = -\frac{2\hbar^2}{m a_1}\delta(x)$

Lieb-Liniger parameter $\gamma = -\frac{2}{n_1\, a_1} > 0$ for repulsion

$g^{(2)}(0) = d\,e(\gamma)/d\gamma$ from ground state energy / particle

Gangardt / Shlyapnikov 2003

measured from photo-association



Kinoshita et al 2005

Lee / Huang / Yang 1957 hard spheres

$g^{(2)}(r \to \sigma^+) = (1 - \sigma/r)^2 + \ldots$ if $n\sigma^3 \ll 1$

• Outline

- **Exact relations for many-body systems with zero-range interactions from the OPE**

  $g^{(2)}(0)$ **is ill-defined and must be replaced by the two-body contact density**

- **Bragg-scattering at large momentum: negative line-shift and multi-particle excitations**

• Zero-range interactions and the OPE

scattering length defined by short distance behavior of

two-body wave function $\psi_0(r) = \frac{1}{r} - \frac{1}{a}$ Bethe / Peierls

many-body problem: separate free motion at short distances

$$\hat{\psi}(\mathbf{R} - \mathbf{x}/2)\, \hat{\psi}(\mathbf{R} + \mathbf{x}/2) = \frac{\psi_0(r)}{4\pi}\, \hat{\phi}(\mathbf{R}) + \ldots$$

Braaten / Platter 2008

OPE $$\mathcal{O}_a(\mathbf{R} - \mathbf{x}/2)\mathcal{O}_b(\mathbf{R} + \mathbf{x}/2) = \sum_\ell W_\ell^{(a,b)}(\mathbf{x})\, \mathcal{O}_\ell(\mathbf{R})$$

• pair distribution function at short separation

$$\hat{n}(\mathbf{R}+\mathbf{x}/2)\,\hat{n}(\mathbf{R}-\mathbf{x}/2) \xrightarrow[|\mathbf{x}|\to 0]{} \delta(\mathbf{x})\,\hat{n}(\mathbf{R}) + \frac{\psi_0^2(r)}{16\pi^2}\,\hat{\phi}^\dagger(\mathbf{R})\hat{\phi}(\mathbf{R}) + \dots$$

contact density $\mathcal{C}_2(\mathbf{R}) = \langle \hat{\phi}^\dagger(\mathbf{R})\hat{\phi}(\mathbf{R})\rangle$

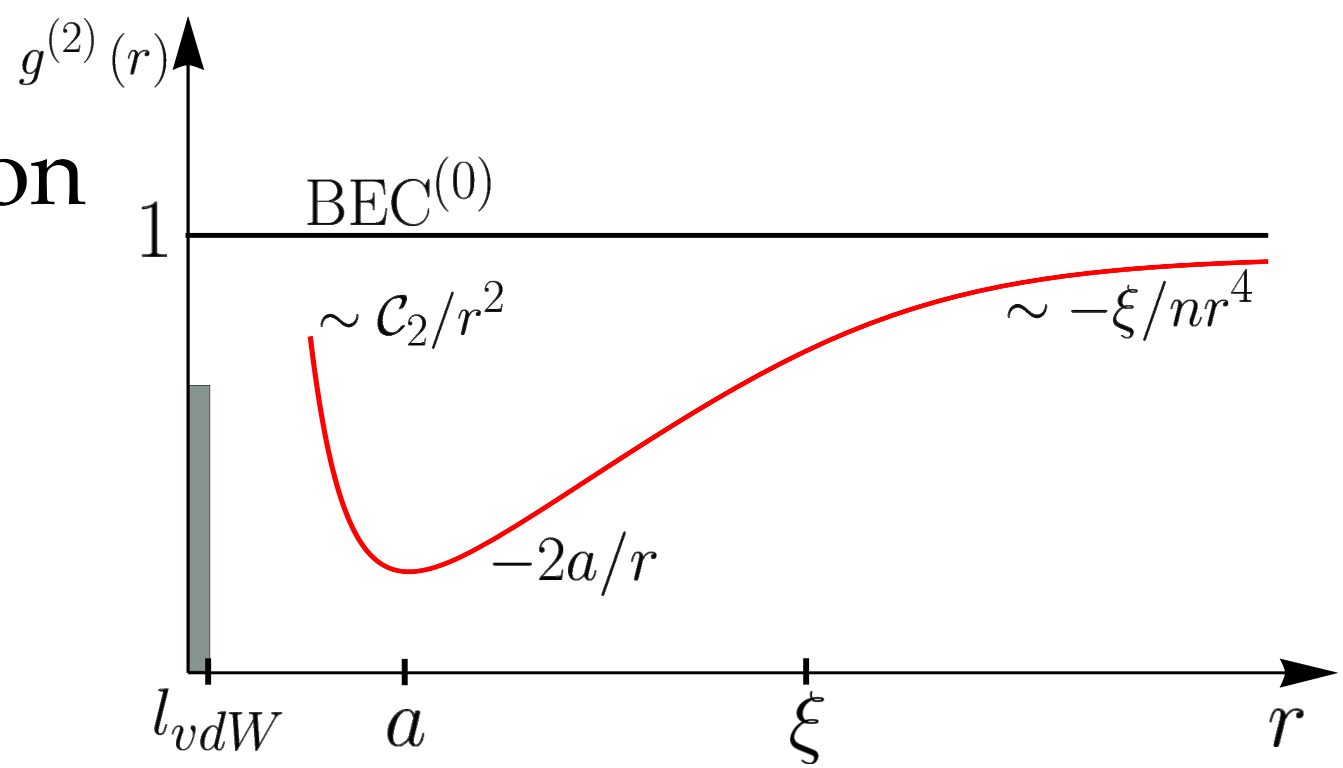
$$\lim_{r\to 0} n^2 g^{(2)}(r) = \frac{\mathcal{C}_2}{16\pi^2}\left(\frac{1}{r^2} - \frac{2}{ar} + \dots\right)$$

S. Tan 2005

BEC with repulsive interaction

bunching at $r \lesssim a$!!

Naraschewski / Glauber 1999



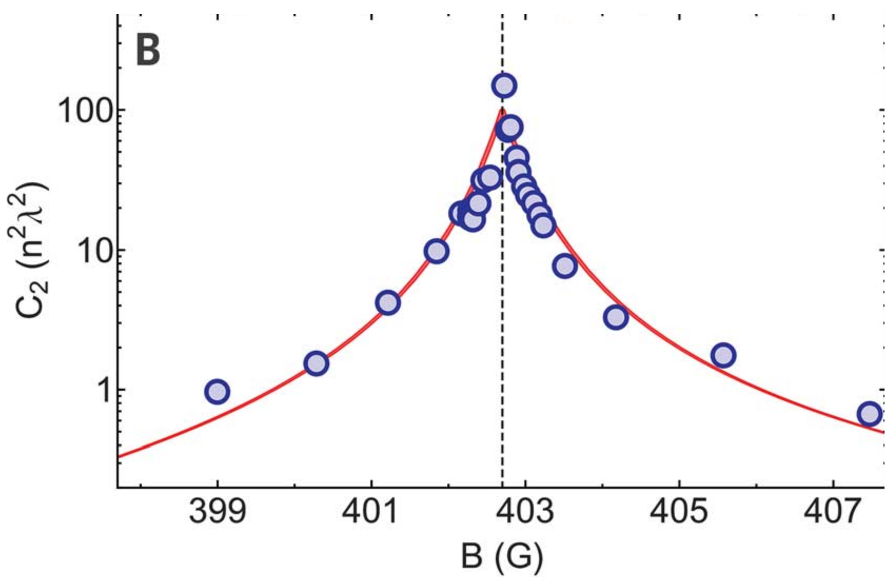
• Tan relations: a list of examples

the two-body contact density determines the

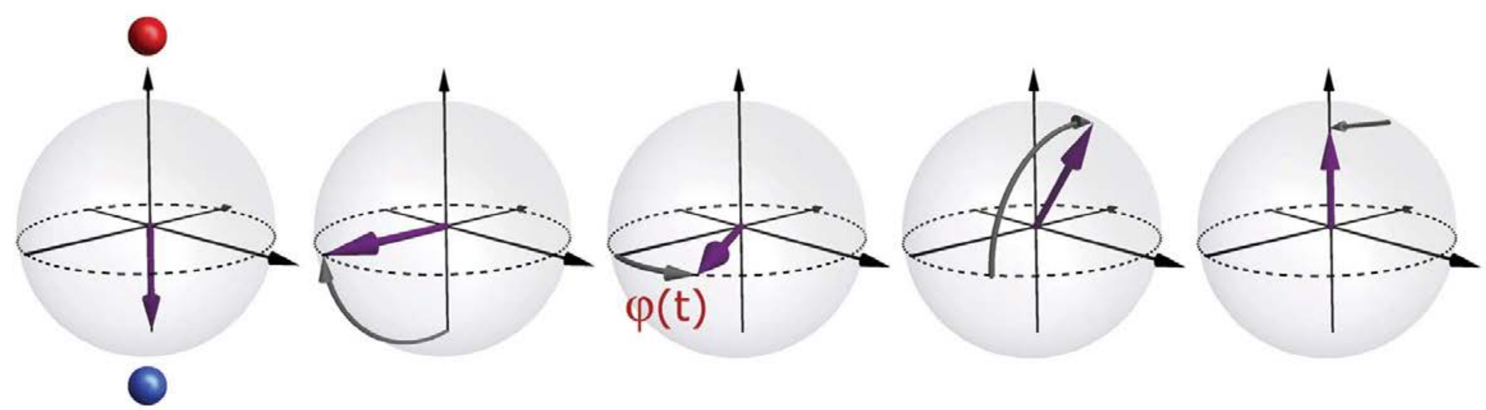
- **tail in the momentum distribution** $n(\mathbf{q}) \to \mathcal{C}_2/q^4 + \ldots$
- **closed channel fraction near a Feshbach resonance**
- **derivative of the energy with respect to 1/a** $\dfrac{\partial\,\varepsilon}{\partial(1/a)} = -\dfrac{\hbar^2}{8\pi m}\mathcal{C}_2$
- **clock-shift and asymptotic decay of the RF-spectrum**
- **rates for two- or three-body losses** $\Gamma_2 = \dfrac{\hbar\,\mathcal{C}_2}{4\pi m\, n}\,\mathrm{Im}\,(1/a)$

• Ramsey-type measurements of the contact

$$\mathcal{C}_2(na^3, T) \rightarrow \begin{cases} (4\pi n a)^2 + \ldots \text{ dilute BEC at } T = 0 \\ 32\pi\,(n\lambda_T)^2 \left(1 - \lambda_T/\sqrt{2}|a| + \ldots\right) \sim 1/T \end{cases}$$



non-degenerate gas with $\lambda_T \ll |a|$

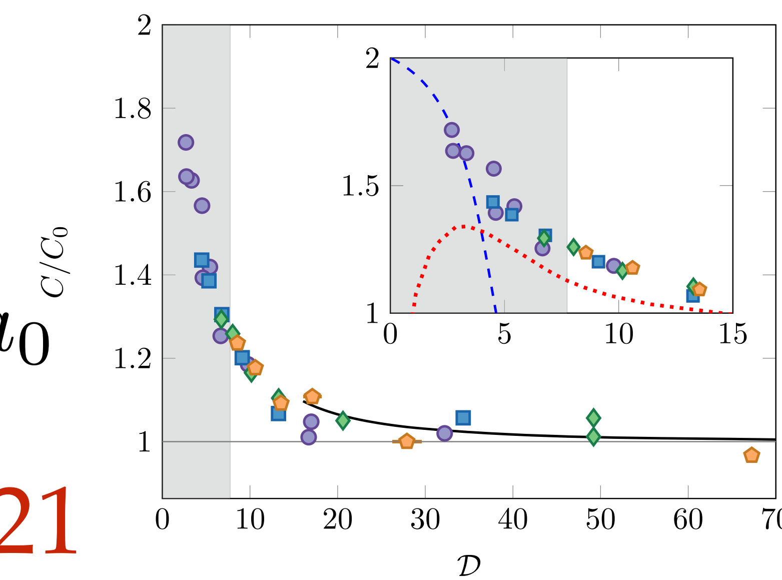


precession rate $\Omega = \frac{\hbar}{4\pi m}\left(\frac{\mathcal{C}_2}{na} + \frac{5\pi^2}{n}\mathcal{C}_3\right)$ Fletcher et al 2017

change in clock-transition frequency ^{87}Rb

$\Delta\nu \sim \mathcal{C}_2 \cdot \Delta a$ with $\Delta a = a_{22} - a_{11} = -5.7\, a_0$

contact across BKT-transition Zou et al 2021



• the Fierz relation for hard spheres

hard sphere fluids $3\,p_{\rm HS} = 2\,\varepsilon + \sigma^2 g''(\sigma)\cdot \pi n^2(\hbar^2\sigma/m)$ Fierz 1957

$\sigma \neq 0$ breaks scale invariance $\longrightarrow$ trace anomaly $\sim g''(\sigma)$

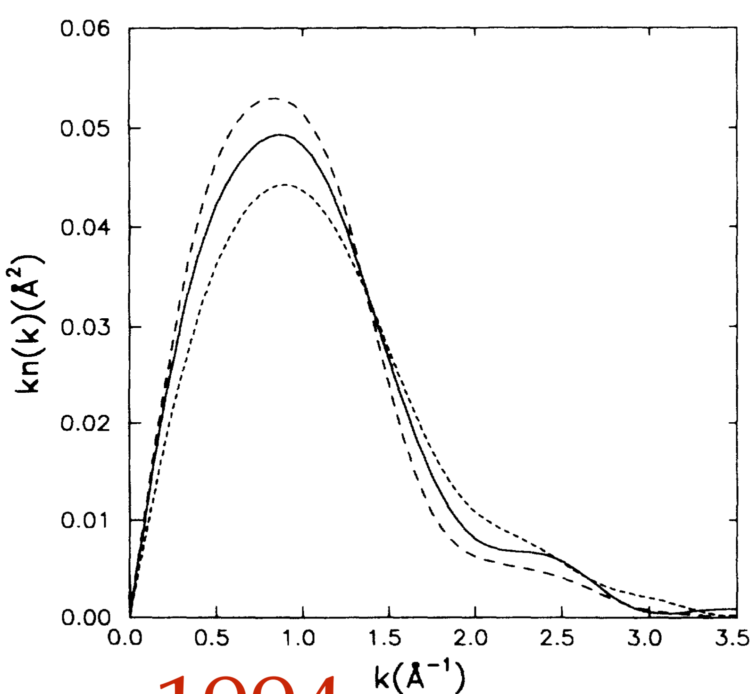
non-analyticity at contact $g^{(2)}(r\to\sigma^+) = g''(\sigma)\,(r-\sigma)^2/2+\dots$

gives rise to $\lim_{q\gg n^{1/3}} n(\mathbf{q}) = \mathcal{C}_{\rm HS}\left(\frac{\sin(q\sigma)}{q^3}\right)^2$ with $2\,\mathcal{C}_{\rm HS} = (4\pi n\sigma)^2 g''(\sigma)$

Tan relation $n(\mathbf{q})\to\mathcal{C}_2/q^4$ emerges for

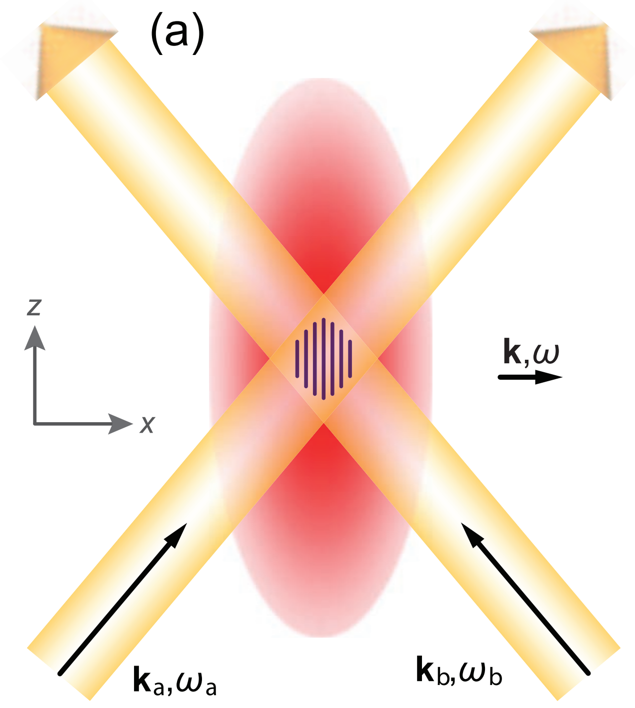
$\sigma\to 0$ and $\sigma^2\mathcal{C}_{\rm HS}\to\mathcal{C}_2$ finite !

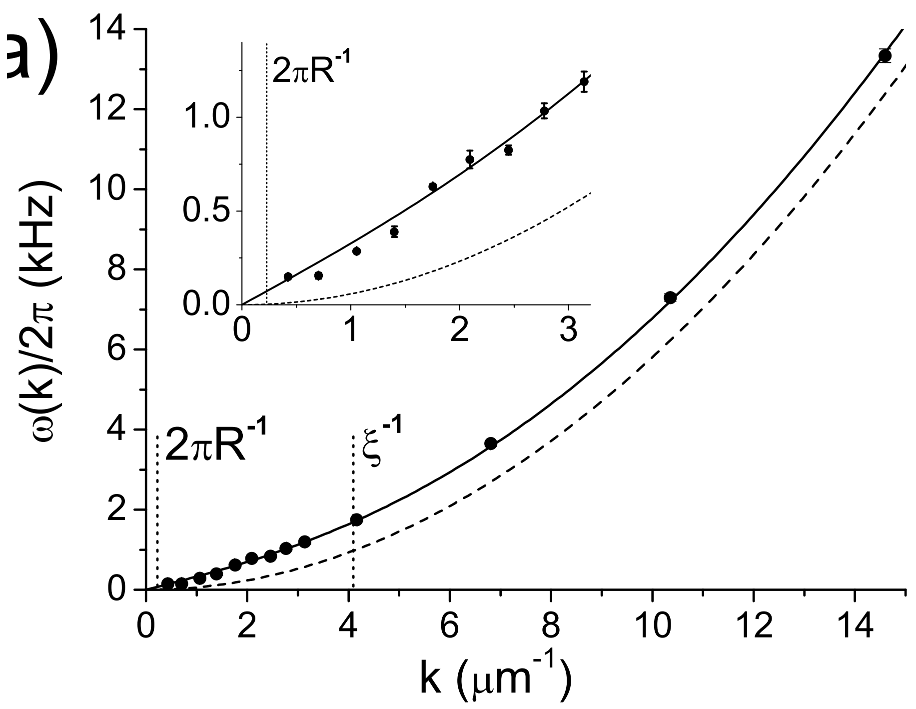
LHY 1957 $\sigma^2 g''(\sigma)\to 2$



Boronat / Casulleras 1994

- **Bragg scattering at large momentum**





C. Carcy et al 2019

J. Steinhauer et al 2002

rate of momentum transfer $\frac{d\mathbf{P}}{dt} = -\frac{\hbar \mathbf{q}\,\Omega^2}{2}\,\mathrm{Im}\chi(\omega + i0, \mathbf{q})$

$$S(\omega, \mathbf{q}) = \frac{1}{\pi}\left[1 - e^{-\beta\hbar\omega}\right]^{-1} \mathrm{Im}\,\chi(\omega + i0, \mathbf{q}).$$

• the single mode approximation (SMA)

Bogoliubov theory $(n^{1/3}a \ll 1)$ $\hat{H}_{\text{Bog}} = E_0 + \sum_q E_q \hat{\alpha}_q^\dagger \hat{\alpha}_q$

$$S_{\text{Bog}}(\omega, \mathbf{q}) = n\, S(\mathbf{q})\, \delta(\hbar\omega - E_{\mathbf{q}}) \quad \text{with} \quad E_{\mathbf{q}} = \varepsilon_{\mathbf{q}}/S(\mathbf{q})$$

R. Feynman 1954 SMA is **exact** at small momenta $q\xi \ll 1$

Bogoliubov spectrum at large momentum $q\xi \gg 1$

$$E_{\mathbf{q}} = \sqrt{\varepsilon_{\mathbf{q}}(\varepsilon_{\mathbf{q}} + 2gn_0)} \quad \text{approaches} \quad E_{\mathbf{q}} = \varepsilon_{\mathbf{q}} + gn + \ldots$$

• Line shift in strongly interacting BEC's

Bragg scattering in $^{85}\mathrm{Rb}$ up to $(qa)_{\mathrm{max}} = 0.8$ Papp et al 2008

line-shift $\Delta(\hbar\omega) = \hbar\tilde{\omega}_{\mathbf{q}} - \varepsilon_{\mathbf{q}} \to gn$ should be **linear** in **a**

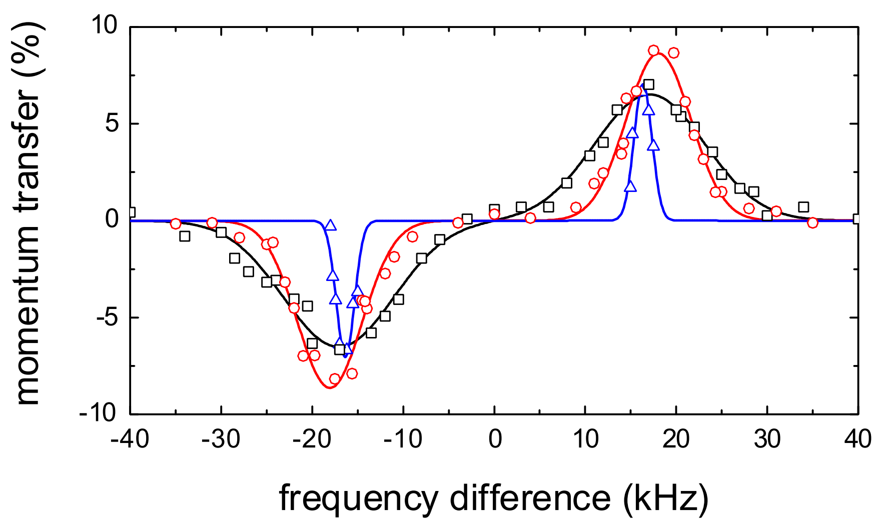
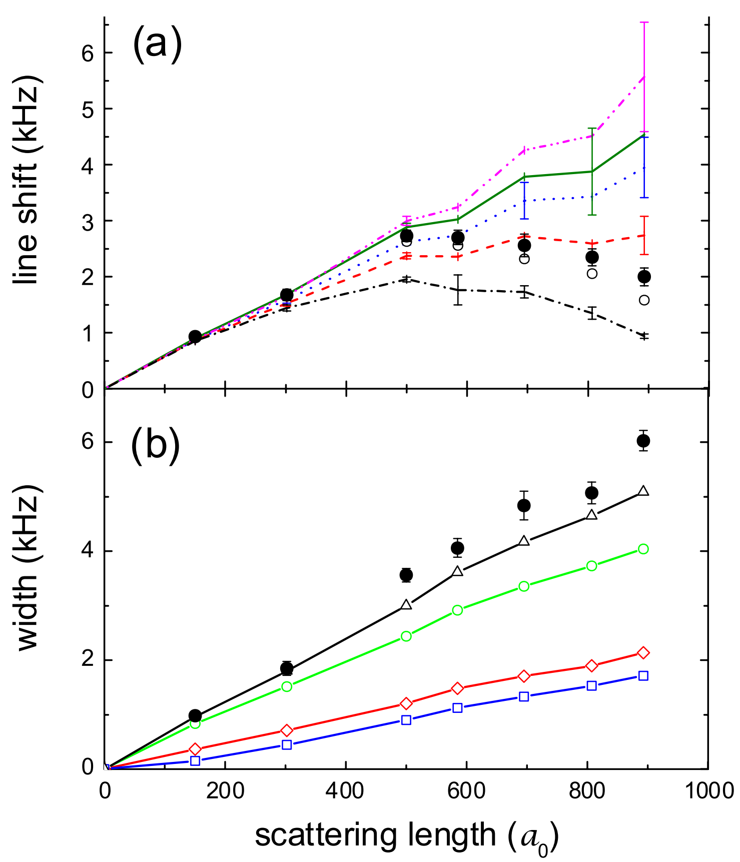


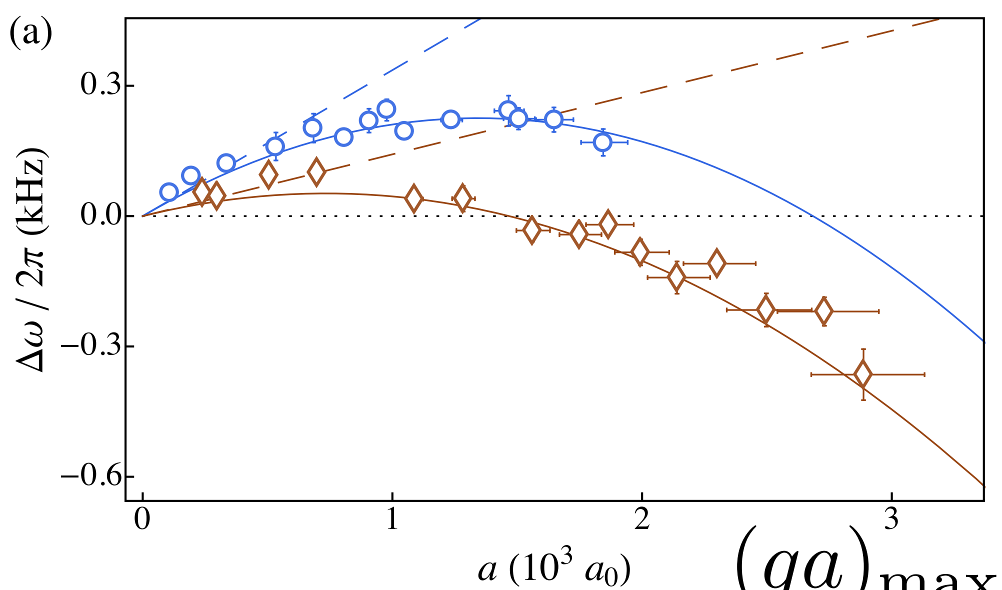
FIG. 2 (color online). Typical Bragg spectra at a scattering length of $100a_0$ (blue triangles), $585a_0$ (red circles), and $890a_0$ (black squares). The excitation fraction is determined from the measured momentum transferred to the BEC and plotted as a function of the frequency difference between the two Bragg beams. Lines are fits of the data as described in the text.



downturn of line-shift near $qa = \mathcal{O}(1)$

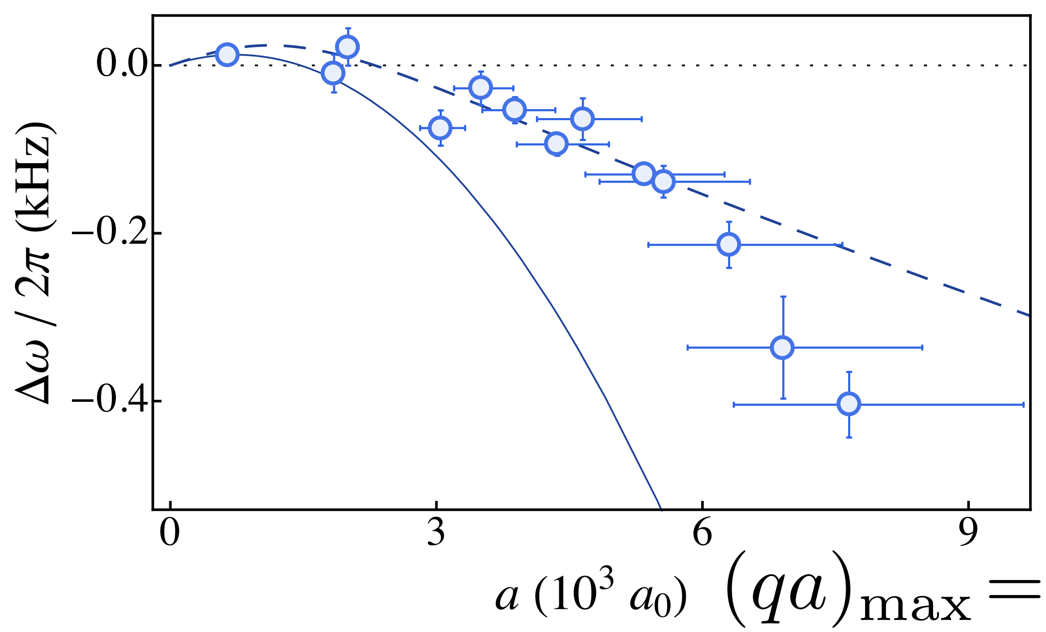
• Bragg scattering on BEC's in a box configuration

Lopes et al 2017 ^{39}K in a box, Feshbach reson. at $B_0 = 402.7\,\mathrm{G}$



$(qa)_{\max} = 2.5$

line-shift



$(qa)_{\max} = 7$

$$\lim_{r\to 0} n^2 g^{(2)}(r) = \frac{\mathcal{C}_2}{16\pi^2}\left(\frac{1}{r^2} - \frac{2}{ar} + \ldots\right) \quad \text{implies} \quad S(q) \to 1 + \frac{\mathcal{C}_2}{8nq}\left(1 - \frac{4}{\pi q a}\right) + \ldots$$

single mode approximation predicts non-monotonic line shift

$$\Delta\left(\hbar\omega_q\right)_{\mathrm{SMA}} = \varepsilon_q\left[S^{-1}(q) - 1\right] \to gn(1 - \pi qa/4 + \ldots)$$

Feynman / Tan

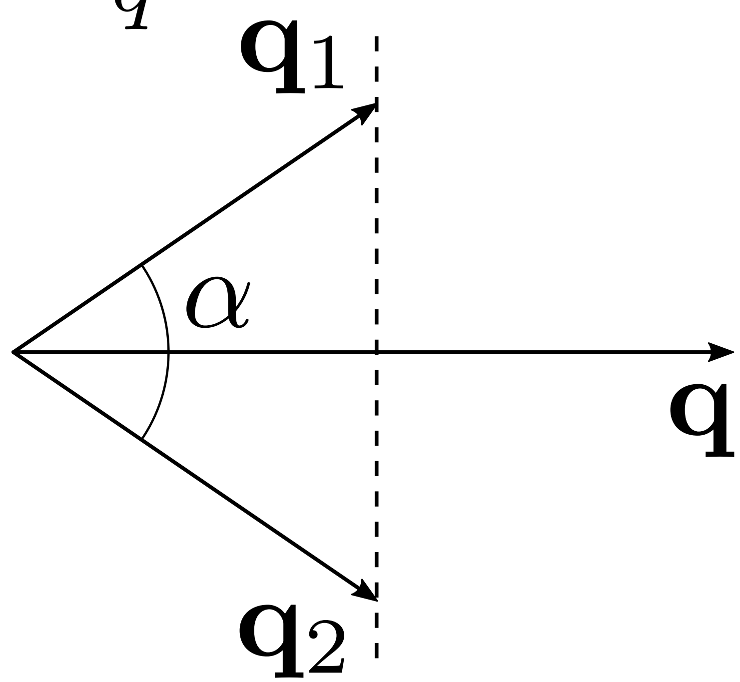
• One-particle versus two-particle excitations

Hohenberg / Platzman 1966 $S_{\mathrm{IA}}(\omega, \mathbf{q}) = \int_{\mathbf{k}} n(\mathbf{k})\, \delta(\hbar\omega + \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{q}})$

gives rise to scaling $S(\omega, \mathbf{q}) = \frac{m}{\hbar^2 \tilde{\xi}^2} \frac{1}{q} J_{\mathrm{IA}}(Y)$ with $Y = \frac{m\tilde{\xi}}{\hbar^2} \frac{\hbar\omega - \varepsilon_{\mathbf{q}}}{q}$

symm. peak around $Y = 0$ line shift $\Delta\left(\hbar\omega_q\right) \sim q$

transfer momentum $\mathbf{q} = \mathbf{q}_1 + \mathbf{q}_2$ to **two** atoms



gives rise to line shifts $\Delta\left(\hbar\omega_q\right) \sim q^2\,!$

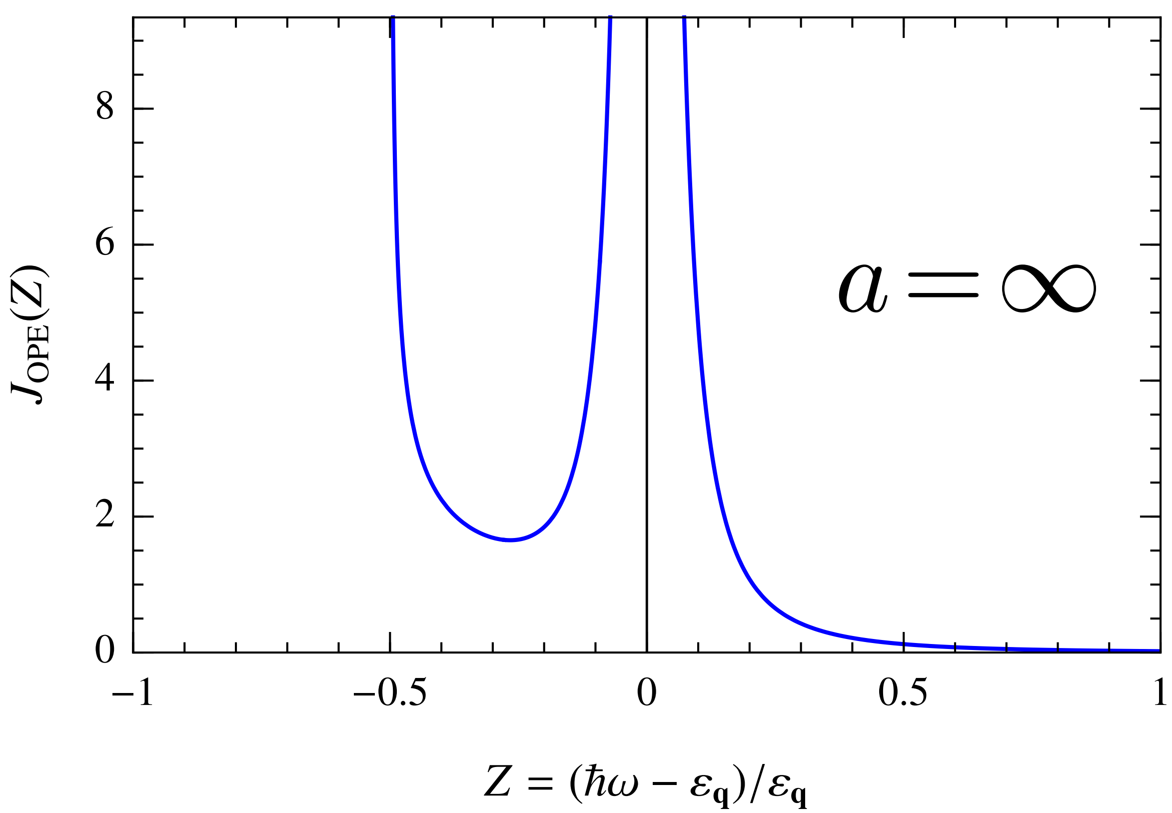
continuum of energies $\varepsilon_{\mathbf{q}}/2 \leq \hbar\omega^{(2)}(\alpha) = \frac{\varepsilon_{\mathbf{q}}}{2\cos^2 \alpha/2} < \infty$

• Dynamical structure factor at large momentum

short-distance expansion $\hat{n}(t,\mathbf{r})\hat{n}(0,\mathbf{0}) = \sum_\ell W_\ell(t,\mathbf{r},a)\hat{O}_\ell(0,\mathbf{0}).$

asymptotic series for $\chi(\omega,\mathbf{q}) = \sum_\ell \frac{m}{\hbar^2 q^{\Delta_\ell - 1}} J_\ell\Big(Z, \frac{1}{qa}\Big)\langle\hat{O}_\ell(0,\mathbf{0})\rangle$

two-particle contribution $S(\omega,\mathbf{q}) = \frac{m\mathcal{C}_2}{\hbar^2 q^3} J_2\left(Z = (\hbar\omega - \varepsilon_\mathbf{q})/\varepsilon_\mathbf{q}\right)$



spectrum is asymmetric !

collinear singularity at $\hbar\omega = \varepsilon_\mathbf{q}/2$

tail $S(\omega,\mathbf{q}) \sim \mathcal{C}_2 q^4/\omega^{7/2}$

Son / Thompson 2010

• Crossover impulse-approximation to OPE

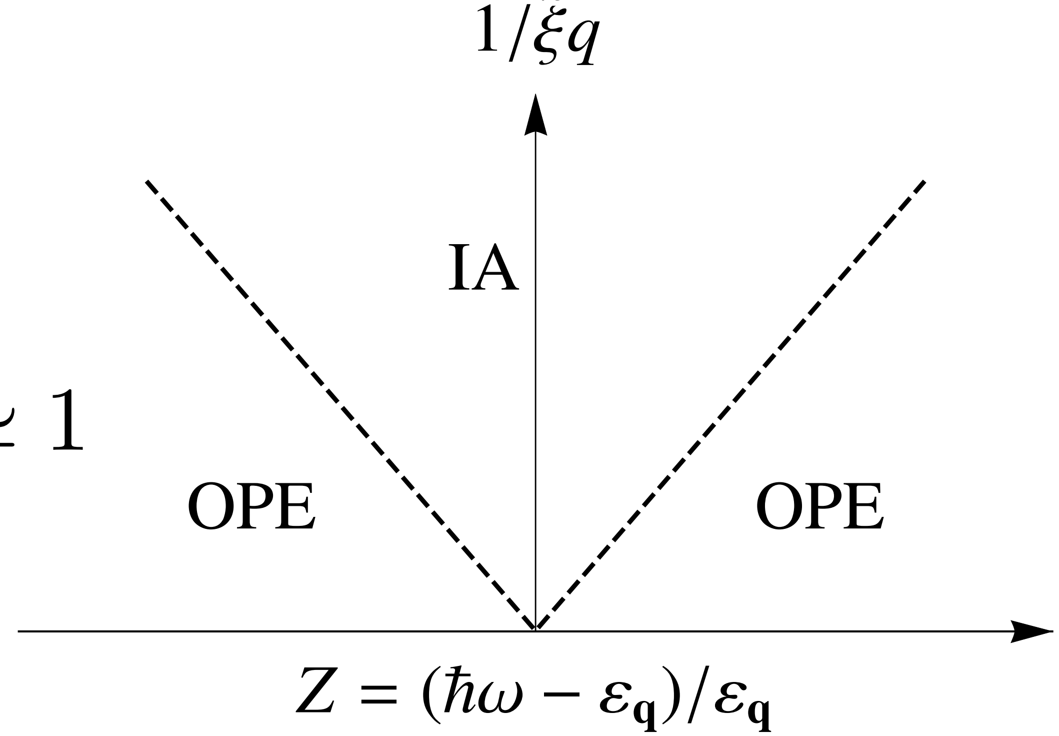
single particle excitations $\hbar\omega = \varepsilon_{\mathbf{k}+\mathbf{q}} - \varepsilon_{\mathbf{k}} \;\rightarrow\; Y = \frac{m\tilde{\xi}}{\hbar^2}\frac{\hbar\omega - \varepsilon_{\mathbf{q}}}{q} = \mathcal{O}(1)$

$$\lim_{|Y|\gg 1} S_{\mathrm{IA}}(\omega,\mathbf{q}) = \frac{m\mathcal{C}_2}{\hbar^2 q^3}\frac{q^2\tilde{\xi}^2}{8\pi^2 Y^2} \qquad \text{due to} \qquad n(k\tilde{\xi}\gg 1) = \mathcal{C}_2/k^4$$

coincides with $\lim_{|Z|\ll 1} S_{\mathrm{OPE}}(\omega,\mathbf{q}) = \frac{m\mathcal{C}_2}{\hbar^2 q^3}\frac{1}{2\pi^2 Z^2}$ since $Z = 2Y/(q\tilde{\xi})$

smooth crossover from single to

many - particle excitations near $|Y| \simeq 1$



Hofmann/Zw. 2017

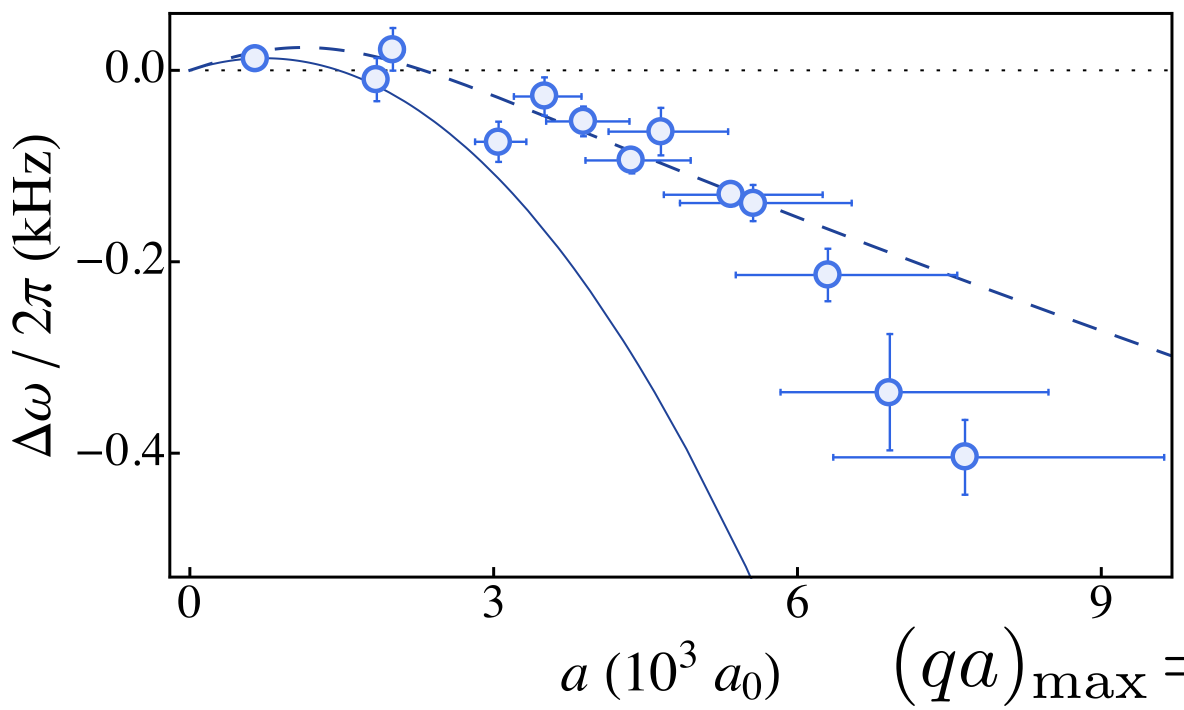
• Shift of the single-particle peak from the OPE

self-energy from OPE $\Pi(\varepsilon_{\mathbf{q}}, \mathbf{q}) = \left[\frac{1}{2\pi a^2}\frac{1}{a^{-1}+iq/2} - \frac{iq}{8\pi} - \frac{1}{4\pi a}\right]\frac{\hbar^2 \mathcal{C}_2}{mn}$

line-shift $\Delta\left(\hbar\omega_q\right) = \mathrm{Re}\,\Pi(\varepsilon_{\mathbf{q}}, \mathbf{q}) = \begin{cases} gn\left(1-(qa)^2/2+\ldots\right) \\ -\frac{\hbar^2\mathcal{C}_2(a)}{4\pi mna} \to 0 \text{ if } qa \gg 1 \end{cases}$

vanishes at $\bar{q}a = 2$ $(\bar{q}a = 4/\pi$ in SMA)

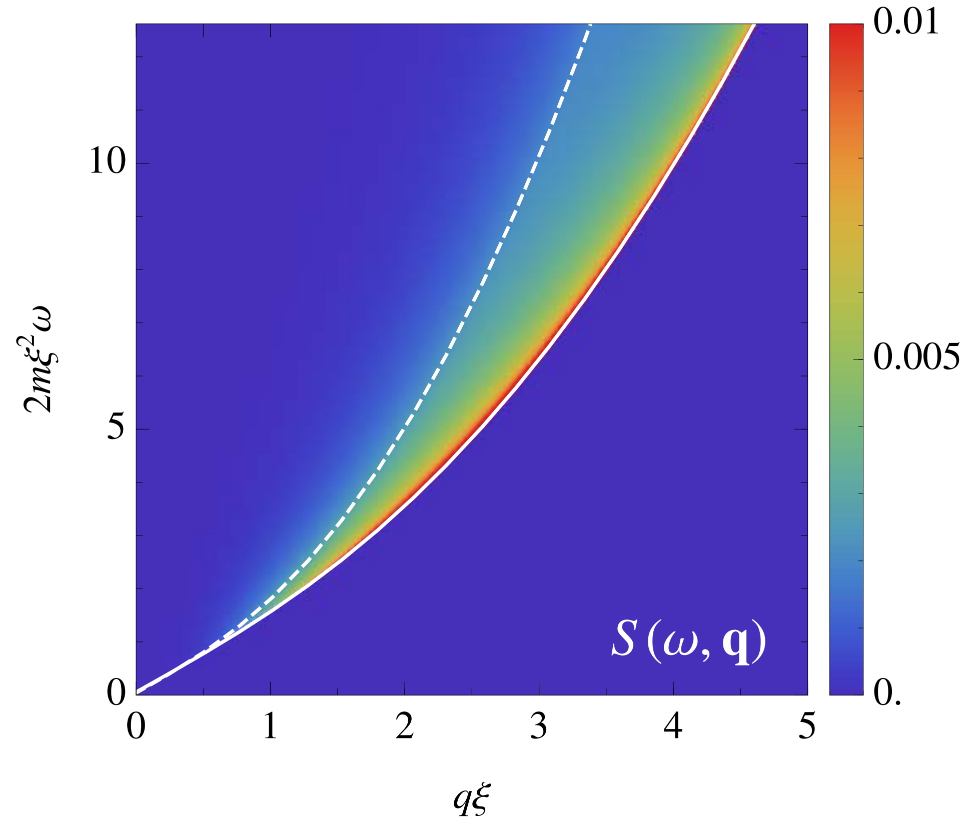
missing: effects of three-body or higher order correlations

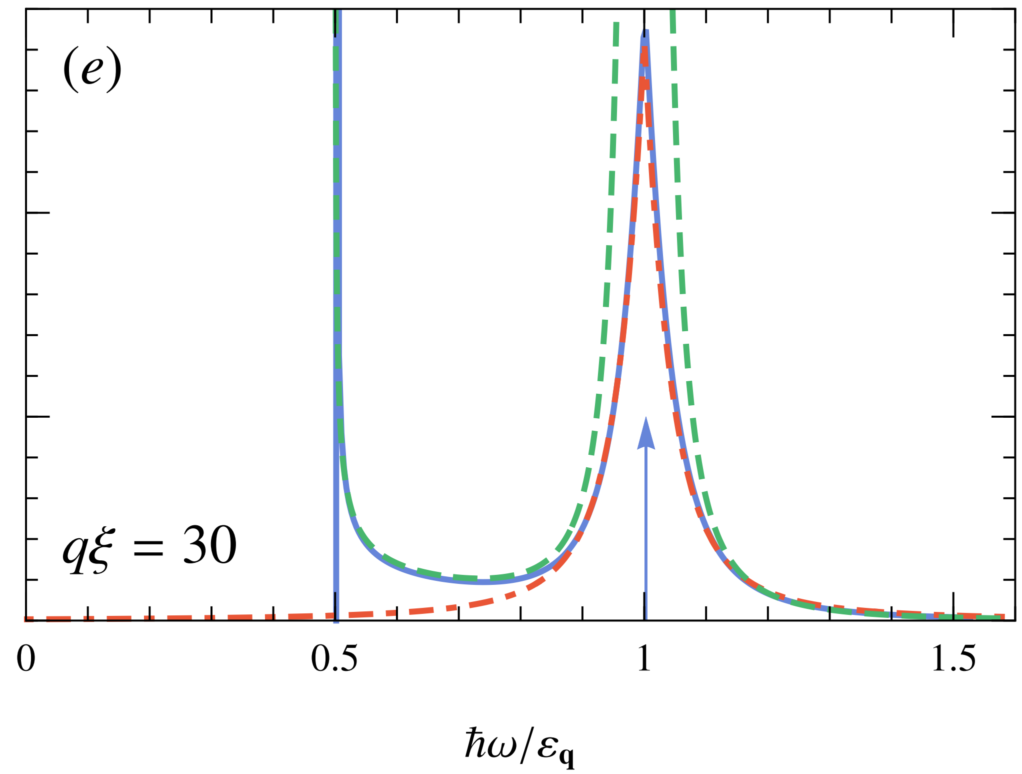


$(qa)_{\mathrm{max}} = 7$

Lopes et al 2017

• Full spectrum within a T-matrix approximation





the spectrum is **not** described by a single-mode approximation

$$\chi(\omega, \mathbf{q}) = -\frac{Z_{\mathbf{q}}}{\hbar\omega - \varepsilon_{\mathbf{q}} - \Pi(\omega, \mathbf{q})} + \chi^{\mathrm{inc}}(\omega, \mathbf{q})$$

incoherent background extends from $\varepsilon_{\mathbf{q}}/n$ with $n = 2, 3 \ldots$ to ∞

• Conclusion

- Quantum fluids with zero range interactions obey a set of exact relations due to S. Tan. They connect short distance or time correlations with thermodynamic properties and they hold for arbitrary states of the many-body problem.

- Bragg scattering at large momentum involves multi-particle excitations. It can be described in a systematic expansion in inverse powers of q. This explains qualitatively the negative line shift observed at JILA in 2008 and in Cambridge 2017. The extension to three-body correlations and an observation of the detailed form of the spectrum remains open.