Losses in quantum gases with contact interactions with an emphasis on the one-dimensional case

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Collège de France, 14$^{\text{th}}$ April 2023

Open quantum systems

Quantum systems coupled to environements :

- Exchange of energy $\Rightarrow$ heat transfer
- Exchange of particles
  - Transport phenomena
  - Case of loss process
    - Zeno effect $\rightarrow$ strongly correlated gases
      F. Verstraete et al., Nature Physics 5, 633 (2009)
      M. Roncaglia et al., Phys. Rev. Lett. 104, 096803 (2010)
      N. Syassen et al., Science 320, 1329 (2008)

Effect of losses : still a lot to discover

Losses in a correlated system : a field at its begining
Here : slow losses in gases with contact interactions

"Long-lived nonthermal states realized by atom losses in one-dimensional quasicondensates" (2017)
"Cooling phonon modes of a Bose condensate with uniform few body losses" (2018)
"Cooling a Bose gas by three-body losses" (2018)
"Asymptotic temperature of a lossy condensate" (2020)
"The effect of atom losses on the distribution of rapidities in the one-dimensional Bose gas" (2020)
"Breakdown of Tan's relation in lossy one-dimensional Bose gases" (2021)
"Losses in interacting quantum gases : Ultraviolet divergence and its regularization" (2021)

Outline

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1. UV divergence in $D > 1$ for 1-body losses
2. Losses in 1D Bose gas : general results
   - Evolution of the rapidity distribution
   - Analytic result for hard-core bosons
3. Failure of Tan's relation
4. Quansicondensate limit
5. Experimental results : asymptotic temperature of phonons

Reservoir of vanishing correlation time : Lindblad equation

Homogeneous 1-body loss process

- System : density matrix ρ

Lindblad equation for homogeneous 1-body losses

- Reservoir of correlation time $\ll$ system's dynamics time

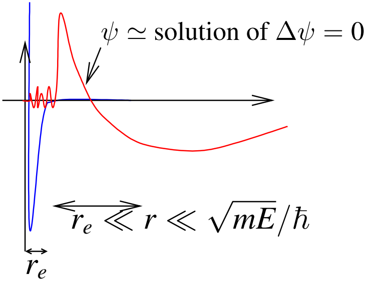
$$\frac{d\rho}{dt} = -i[H,\rho] + \Gamma \int d^d\mathbf{r} \left(-\frac{1}{2}\{\psi_{\mathbf{r}}^+\psi_{\mathbf{r}}, \rho\} + \psi_{\mathbf{r}}\rho\psi_{\mathbf{r}}^+ \right)$$

Lindblad equation : used in all previous works in cold atoms domain

- **Universal behavior** : single parameter Γ
- **trivial if uncorrelated system** $\rho = \prod_a \rho_a : dN_a/dt = -\Gamma N_a$
- If quantum correlations present between atoms ?
  $\Rightarrow$ Difficult problem

Interactions between atoms $\Rightarrow$ correlations

Interactions between atoms : contact interaction



Potential $\Rightarrow$ boundary condition for $r \to 0$

For $r_e \ll r \ll \sqrt{mE}/\hbar$

$\psi \propto 1/r - 1/a$ (3D)

$\psi \propto \ln(r/a)$ (2D)

$\psi \propto 1 - r/a$ (1D)

Single parameter : scattering length a

Universal behavior

Pseudo-potential : $V = \delta^d(\mathbf{r})U$

$1/p^4$ momentum tail tail $\Rightarrow$ UV divergence of E_{kin}

For $p^2/m \gg E$, $\boxed{|\psi_p|^2 = \frac{m}{L^d}\,(U\psi)^2_{r=0}\,\frac{1}{p^4} = \frac{\alpha}{\mathbf{p}^4}}$

$E_{\text{kin}} \propto \int d^d\mathbf{p}\,\frac{p^2}{2m}|\psi_p|^2 \Rightarrow E_{\text{kin}} = \infty$ for $D > 1$

compensated by diverging interaction energy (E_{tot} finite)

UV divergence of the energy deposited by a loss event

Lindblad : loss event instantaneous

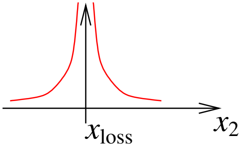
$$\Rightarrow |\psi\rangle \to |\psi'\rangle = \psi_{\mathbf{r}_{\rm loss}}|\psi\rangle \Rightarrow \psi'(\mathbf{r}_2, \mathbf{r}_3, \dots) \propto \psi(\mathbf{r}_1 = \mathbf{r}_{\rm loss}, \mathbf{r}_2, \mathbf{r}_3, \dots)$$

Singularity at loss event position :

$\Rightarrow$ momentum tails decreasing as $1/p^4$

$\Rightarrow E_{\rm kin} = \infty$

No compensation by interaction term



Energy diverges in $D > 1$ after a loss event

UV catastrophe for Lindblad dynamics with contact interactions

$$dE/dt = \infty$$

Regularisation

- Finite interaction range
- Finite correlation time of the reservoir $\Rightarrow$ finite energy width

Reservoir of finite energy width E_{res}

Role of the contact

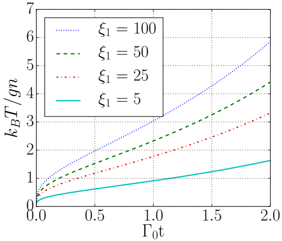
- Contact C : quantifies the **number of pairs** in the system
- **Amplitude of** $1/p^4$ **momentum tails** :$C = \lim_{p\to\infty} p^4 n(p)$

Energy increase rate

For large E_{res}, $dE/dt = \Gamma_0 \frac{C}{m} \mathcal{B}$ with $\mathcal{B} \simeq \sqrt{mE_{\text{res}}}$ (3D)

Case of a BEC : Bogoliubov calculation

Slow losses $\Rightarrow$ at any time, thermal equilibrium at temperature $T(t)$



$$\xi_1 = \sqrt{E_{\text{res}}/(gn_0)}$$

I. Bouchoule et al., arxiv (2021)

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K-body losses in a 1D Bose gas with Contact repulsive interactions

Reservoir of large energy width : Lindblad equation

$$\frac{d\rho}{dt} = -i[H,\rho] + G\int dx\left(-\frac{1}{2}\{\psi_x^{+K}\psi_x^K,\rho\} + \psi_x^K\rho\psi_x^{+K}\right)$$

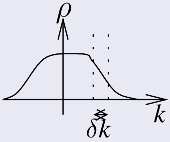
$$H = \int dx\,\left(-\hbar^2/(2m)\psi_x^+\partial_x^2\psi_x + (g/2)\psi_x^{+2}\psi_x^2\right)$$

Integrable system : No relaxation towards thermal equilibrium
Keeping track of n and e not sufficient

Rapidity distribution

Eigenstates of Lieb-Liniger :
$\psi(x_1 < x_2 < \cdots < x_N) = \sum_\sigma a_\sigma e^{i(k_{\sigma(1)}x_1 + k_{\sigma(2)}x_2 + \cdots + k_{\sigma(N)}x_N)}$
$k_1, \ldots, k_N$: rapidities

Rapidity distribution



$$\rho(k)\delta k = \frac{\#\text{ rapidities in } [k, k+\delta k]}{L}$$

Relaxation

System locally described by $\rho(k)$

Slow losses : At each time, system described by $\rho(k,t)$
effect of losses :

$$\boxed{\frac{d\rho(k)}{dt} = -Gn^{K-1}F[\rho](k)}$$

Evolution of $\rho(k)$

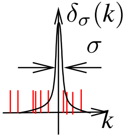
Evolution of conserved quantities under Lindblad

$Q = \int dx q(x)$ $\qquad$ $d\langle Q\rangle/dt = LG\langle \psi_0^{+K}[Q, \psi_0^K]\rangle_{[\rho]}$

Observable Q_k :

$Q_k|\{\lambda_\nu\}\rangle = \sum_\nu \delta_\sigma(k-\lambda_\nu)|\{\lambda_\nu\}\rangle$

$1/L \ll \sigma \ll (d\rho(k)/dk)/\rho(k) \Rightarrow \boxed{L\rho(k) \simeq \langle Q_k\rangle}$



$$F[\rho](k) = -n^{1-K} \sum_{\{\lambda_i\}} p_{\{\lambda_i\}} \sum_{\{\mu_j\}} |\langle\{\mu_j\}|\Psi^K(0)|\{\lambda_i\}\rangle|^2 \left(\sum_j \delta_\sigma(\mu_j - k) - \sum_i \delta_\sigma(\lambda_i - k)\right)$$

Numerical calculation (J. Dubail) :

- Double Markov chain
- Form factors : L. Piroli and P. Calabrese (2015), B. Pozsgay (2011)

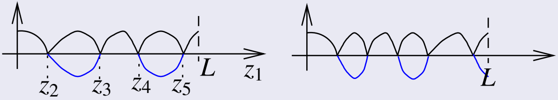
Exact results in the hard-core Bosons limit

Hard-core Bosons : Typical energy per atom fulfills : $E \ll mg^2/\hbar^2$.
$\langle \psi^{+K} \psi^K \rangle = 0$ if $K > 1$: $K = 1$ only relevant

Mapping to fermions : Jordan-Wigner $c(x) = (-1)^{N_{[0,x]}} \Psi(x)$

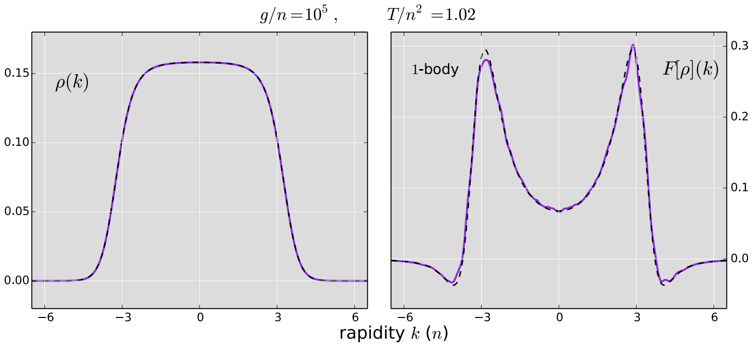
Fermionic boundary condition : depends on N parity

N odd : periodic | N even : antiperidoic



$$F[\rho](k) = \rho(k) - 2\pi \left(\rho(k)^2 - \left(\frac{1}{\pi} \fint \frac{\rho(\lambda) d\lambda}{k - \lambda} \right)^2 \right) + \frac{2n}{\pi} \fint \frac{\rho(k) - \rho(\lambda)}{(k-\lambda)^2} d\lambda$$

Fonctional $F[\rho]$ in the hard-core Bosons limit



Exact time integration : analytic solution for $\rho(k,t)$

losses $\Rightarrow$ $\rho(k,t)$ non thermal

Numerical results in very good agreement with exact result

I.Bouchoule, Benjamin Doyon, Jerome Dubail, SciPost Phys. **9**, 044 (2020)

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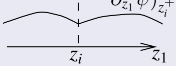
Breakdown of Tan's relation

The Tan's relation : a famous thermodynamic relation

Contact interactions : velocity distribution $\lim_{k\to\infty} k^4 w(k) = c_c$

$$c_c = m^2 g^2 n^2 g_2(0)/(2\pi\hbar)$$

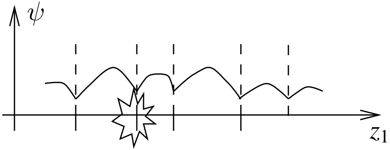


$$\partial_{z_1}\psi)_{z_i^+} - \partial_{z_1}\psi)_{z_i^-} = g\psi(z_1 = z_i, \dots)$$

Olshanii and Dunjko (Phys. Rev. Lett. 91, 090401 (2003).).

Generalized Tan's relation for rapidity distribution with $1/k^4$ tails

$$\lim_{k\to\infty} k^4 w(k) = c_c + c_r, \qquad \text{where } c_r = \lim_{k\to\infty} k^4 \rho(k)$$



Initial growing of $1/k^4$ tails of $\rho(k)$:

$$\frac{dc_r}{dt} = G n^{K+1} K^2 g^2 g^{(K)}(0) g^{(K+1)}(0)$$

Conclusion : Tan's relation most probably violated in 1D

I Bouchoule, J Dubail, Physical Review Letters **126** (16), 160603 (2021)

Outline

Quasi-condensate : Bogoliubov description

Wavelength considered : $\gg 1/n$. Small density fluctuations

Bogoliubov Hamiltonian

Linearisation in $\delta n(\mathbf{r})$ and $\theta(\mathbf{r})$
Collective modes : Fourier modes. $H = \mathrm{Cste} + \sum_{\mathrm{k}} \mathrm{H_k}$

$$H_k = \frac{\hbar\omega_k}{2}\left(\frac{1}{2n_0} f_k \delta n_k^2 + 2n_0 f_k^{-1}\theta_k^2\right) = \hbar\omega_k(a_k^+ a_k + 1/2)$$

$$\omega_k = 2\sqrt{\hbar^2k^2/(2m)(\hbar^2k^2/(2m)+2gn)}, \quad f_k = \hbar\omega_k/(\hbar^2k^2/(2m))$$

Effect of slow 1-body losses on Bogoliubov modes :
P. Grisins et al., Phys. Rev. A93, 033634 (2016)
Our work :

- quantum trajectories calculation, K-body losses, trapped gases

Effect of losses

Effect on density fluctuations and phase flluctuations

$$d\delta n = \underbrace{-K^2 G n_0^{K-1} \delta n\, dt}_{\text{Cooling}} + \underbrace{d\eta}_{\text{shot noise heating}},\ \langle d\eta(\mathbf{r}) d\eta(\mathbf{r}')\rangle = K^2 G n_0^K \delta(\mathbf{r}-\mathbf{r}')dt$$

If lost atom number recorded : increase of knowledge on δN
$\Rightarrow \langle \delta N^2 \rangle \searrow \Rightarrow \langle \theta^2 \rangle \nearrow$

$$\boxed{\langle d\theta(\mathbf{r}) d\theta(\mathbf{r}')\rangle = \frac{K^2}{4} G n_0^{K-2} \delta(\mathbf{r}-\mathbf{r}')dt}$$

Fourier representation

$$\frac{d}{dt}\langle \delta n_k^2 \rangle = -2K^2 G n_0^{K-1} \langle \delta n_k^2 \rangle + K^2 G n_0^K$$

$$\frac{d}{dt}\langle \theta_k^2 \rangle = K^2 G n_0^{K-2}/4$$

Evolution of Bogoliubov modes

Mode population

$$\frac{d\langle a_k^+ a_k\rangle}{dt} = K^2 G n^{K-1}\left(-\langle a_k^+ a_k\rangle - 1/2 + (f_k + f_k^{-1})/4\right)$$

Large p behavior : $d\langle a_p^+ a_p\rangle/dt = K^2 G n^{K-1}\left(-\langle a_p^+ a_p\rangle + m^2 g^2 n^2/p^4\right)$

$\rightarrow 1/p^4$ tails

Phononic limit $\hbar^2 k^2/m \ll gn = mc^2$, $y = \hbar\omega_k \langle a_k^+ a_k\rangle/(mc^2)$

$$dy/dt = KGn^{K-1}\left(-y(K-1/2) + K/2\right)$$

Stationnary value : $y_\infty = 1/(2 - 1/K)$

Asymptotic phonons temperature : $k_B T = mc^2 y_\infty$

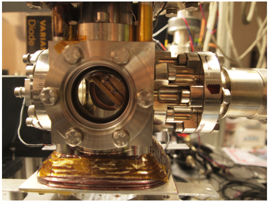
- Calculation extended for non-homogeneous gases (effect of trap)
- Calculation extended to take into account 3D effect at large n

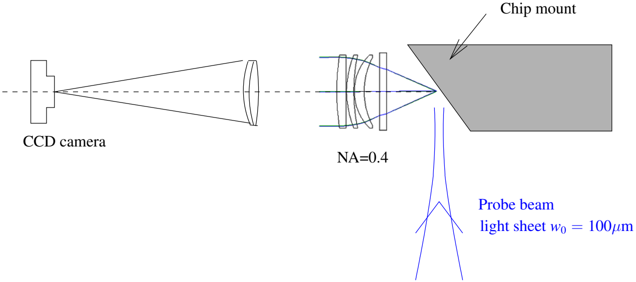
I. Bouchoule et al. SciPost Phys. 5, 043 (2018)

Outline

Experimental setup : realising and imaging 1D gases

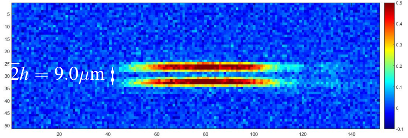
- Magnetic confinement using atom-chip setup





- $N_{\rm at} = 3 - 10 \times 10^3$
- $\omega_z = 8 - 15$ Hz
- $\omega_\perp = 1.5 - 3$ kHz
- $\mu \simeq T = 50 - 100$ nK
- $l_c/\xi \simeq 10$: deep into quasi-BEC

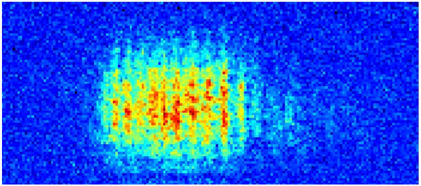
A typical in-situ absoprtion image



Thermometry in qBEC regime via density ripples analysis

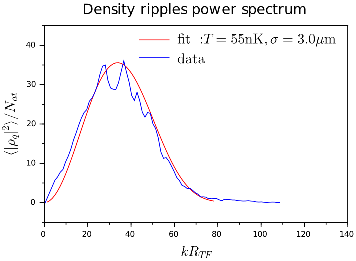
- **Trapping potential suddenly turned off** : interactions $\searrow 0$
- **8 ms time of flight :** $\theta(x) \to$ density fluctuations (density ripples)

Single shot image



Statistical analysis on $\simeq 50$ images

$\Rightarrow$ extract power spectrum $\langle |\rho_q|^2 \rangle$



Fit to deduce temperature

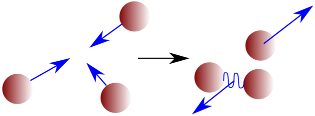
Sensitive to phononic modes

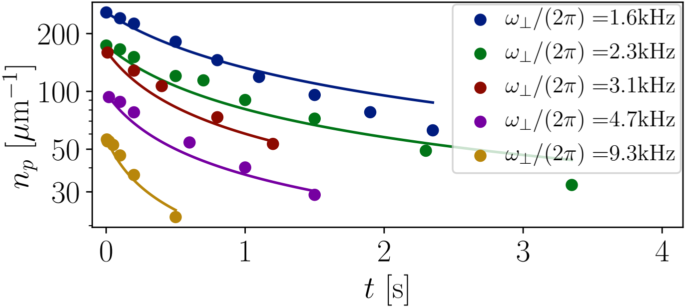
$\Rightarrow$ phonons temperature

Decay of atom number under 3-body loss process

- Losses dominated by 3-body process

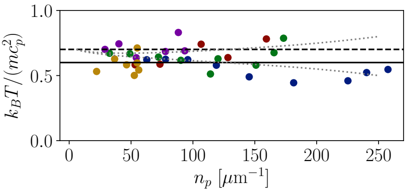
$$\frac{dn}{dt} = -K_3 n^3$$





Evolution of phonons temperature

Evolution of $T/(mc^2)$

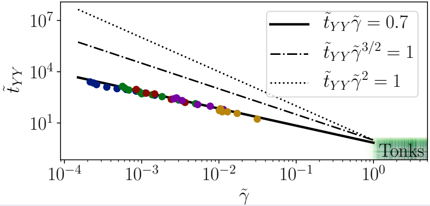


—— : $y_\infty = 0.6$ for homogeneous gas

- - - : $y_\infty = 0.70$ for harmonic trap

Evolution in phase space

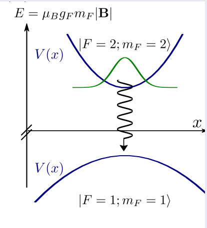
$$\tilde{t} = \hbar^2 k_B T n^2/(m^3 c^4)$$
$$\tilde{\gamma} = m^2 c^2/(\hbar^2 n^2)$$

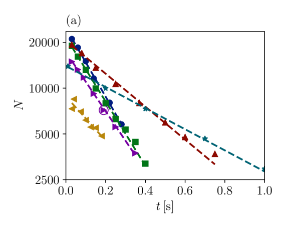


- Good agreement with theory
- Initial state already close to asymptotic behavior

1-body losses

Ingeneered 1-body losses



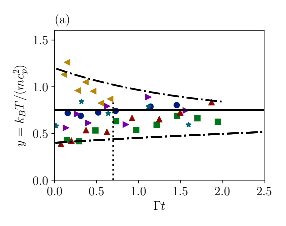


Energy-insensitive losses : noisy HF field

Large parameter range explored :

$\omega_\perp = 1.5 - 4\text{kHz}$

Evolution of the phonons temperature

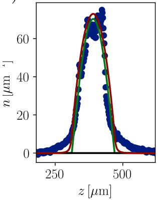


- Good agreement with theory
- Solve the open question raised by former results (Rauer et al. , Phys. Rev. Lett.116, 030402 (2016))

Evidence for non-thermal states produced by losses

- Profile and density ripples incompatible

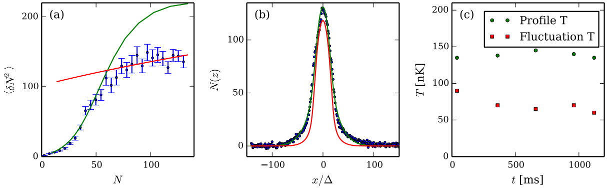
$b)$



Thermal profile at T deduced from density ripples analysis

- Profile and insistu density fluctuations incompatible

Insitu density thermometry : $\langle \delta N^2 \rangle = \frac{k_B T}{\Delta \partial n / \partial \mu}$



A. Johnson et al., Phys Rev A **96**, 013623 (2017)

Conclusion

Results

- UV divergence for Lindblad dynamics with contact interactions in $D > 1 \Rightarrow$ finite E_{res} and/or finite r_e
- Lieb-Liniger gas : evolution of rapidity distribution, analytic expression for har-core bosons
- Peculiar state : failure of Tan's relation
- Observation of the asymptotic temperature of phonons

Prospects

- Numerical effort requiered to compare to profile data
- Link between Bogoliubov picture and $\rho(k)$
  - Bouchoule, et al., Phys Rev Lett (2023)
- Effect of finite reservoir energy width in $D > 1$