

# One-dimensional Yb gases with strong two-body losses

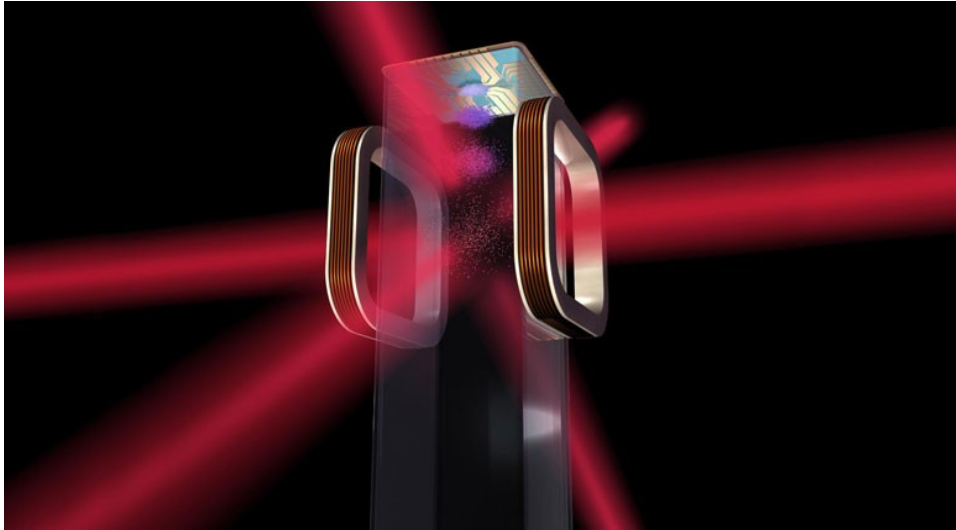
Leonardo Mazza

Université Paris-Saclay, LPTMS

Mini-workshop “Open systems in quantum many-body physics”  
Collège de France – April 14, 2023



# Coherence and decoherence in quantum gases



Gases of bosons or fermions cooled to quantum degeneracy

- No disorder
- Isolated from environments
- Controlled unitary dynamics

# Coherence and decoherence in quantum gases



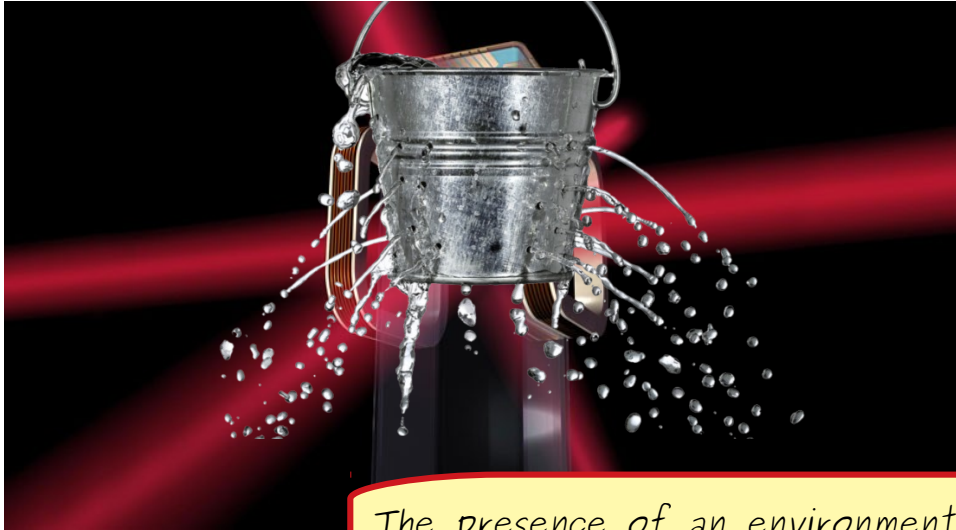
**Gases of bosons or fermions cooled to quantum degeneracy**

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*Yet, particle losses have always been a problem (and still are!)*

*In reality, the quantum gas is coupled to an environment*

# Coherence and decoherence in quantum gases



## Gases of bosons or fermions cooled to quantum degeneracy

- No disorder
- Isolated from environments
- Controlled unitary dynamics

*Yet, particle losses have always been a problem (and still are!)*

*In reality, the quantum gas is coupled to environment*

*The presence of an environment causes decoherence and hides genuine quantum effects*

## Today's talk:

*can we change this viewpoint and find situations where losses produce interesting physics?*



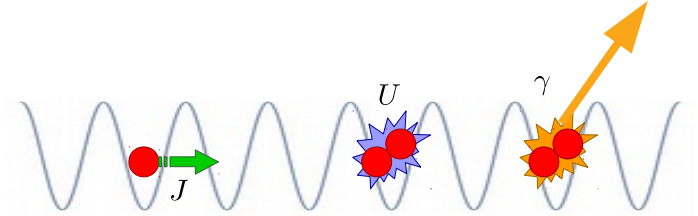
# Losses and engineered correlations

**The problem:** a one-dimensional bosonic gas with strong two-body losses

**First experiment:** Rempe 2009 (MPQ-Munich)

1D optical lattice with Rb-Rb Feshbach molecules (bosons)

Gas prepared with 1 molecule per site. The optical lattice is lowered, what happens?



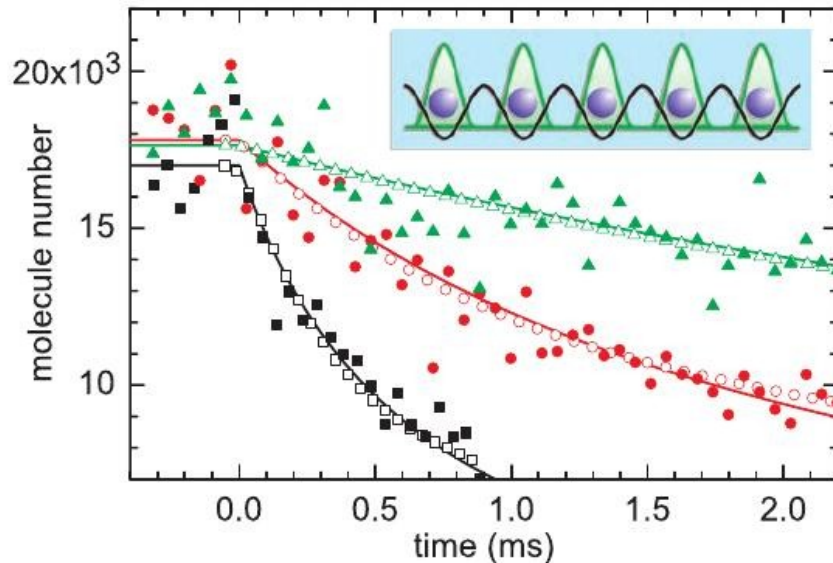
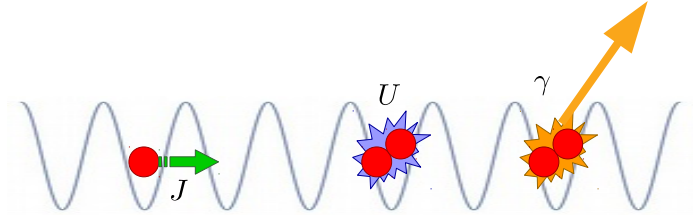
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*By increasing the loss rate beyond a certain threshold, the gas depopulates more slowly. Emergence of a novel decay time.*

**Theory:** quantum Zeno effect (works by Garcia-Ripoll and Cirac, 2009)

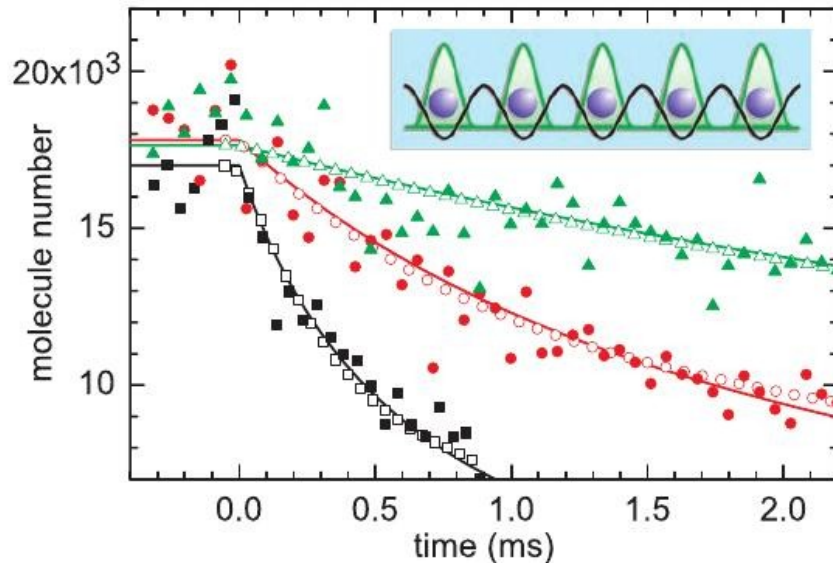
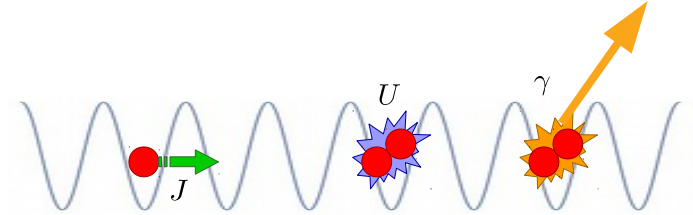
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**Theory:** quantum Zeno effect (works by Garcia-Ripoll and Cirac, 2009)

**The open problem:** to characterise completely the dynamics of a gas subject to two-body losses and in particular the interplay between losses and quantum dynamics

A theoretical activity ignited by discussions @CdF with Fabrice Gerbier and Jérôme Beugnon before the pandemics :)

# Interesting or not interesting?

**My focus:** two-body losses

Mean-field / phenomenological analysis

$$\frac{dn(\tau)}{d\tau} = -\kappa n^2(\tau)$$



The trap empties completely

$$n(\tau) = \frac{n(0)}{1 + \frac{\kappa\tau}{n(0)}} \sim \frac{1}{\tau}$$



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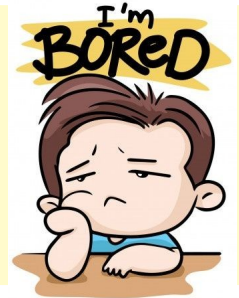
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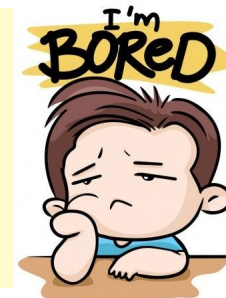
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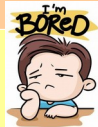
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Our results for one-dimensional lattice models:	Bosons	Fermions
Weak dissipation	We don't expect anything fancy here (but great results in the continuum! Bouchoule, Dubail...) 	Dissipative preparation of entangled Dicke states
Strong dissipation (Zeno)	Anomalous transient dynamics and creation of quantum correlations	Dissipative spin cooling and creation of quantum correlations

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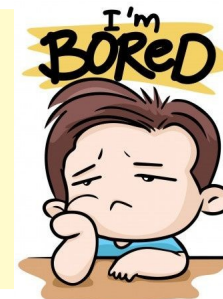
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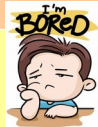
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The stationary states are trivial but they are reached in a non-trivial way

The stationary states are not trivial



# Outline

- **Part 1:** Two-body losses in correlated bosonic gases
- **Part 2:** Two-body losses in correlated fermionic gases
- **Conclusions**

# A master equation for strong two-body losses

The 1D Bose-Hubbard Hamiltonian with parameters J and U

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \gamma \sum_j b_j^2 \rho(t) b_j^{\dagger 2} - \frac{1}{2} \left( b_j^{\dagger 2} b_j^2 \rho(t) + \rho(t) b_j^{\dagger 2} b_j^2 \right)$$

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## Time-scales of the problem

$\frac{1}{\gamma}$  Decay time of  
— double occupancies  
or more

$\frac{\hbar}{J}$  Typical time of the  
— hopping dynamics

**Idea:** use this separation of timescales to develop an effective *perturbative* approach

Perturbative parameter:  $J/(\hbar\gamma)$

- 1) For  $J=0$  all hard-core bosons states are stable (no decay)
- 2) A small value of  $J$  gives to the hardcore bosons a little decay width

$$\Gamma \propto \frac{J^2}{\gamma}$$

# A master equation for hard-core bosons

- Derivation of an effective master equation restricted to the steady space (no double occupancies or more)
- Introduce spin operators

$$H_1 = -J \sum_j \sigma_j^+ \sigma_{j+1}^- + \sigma_{j+1}^+ \sigma_j^-$$

$$C_j = \sqrt{\Gamma_{\text{eff}}} \sigma_j^- (\sigma_{j+1}^- + \sigma_{j-1}^-)$$

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**New emergent time scale**

$$\Gamma_{\text{eff}} = \frac{J^2}{\hbar\gamma} \frac{2}{1 + \left(\frac{U}{\hbar\gamma}\right)^2}$$

## Time-scales of the problem

$\frac{1}{\gamma}$  Decay time of double occupancies or more

$\frac{\hbar}{J}$  Typical time of the hopping dynamics

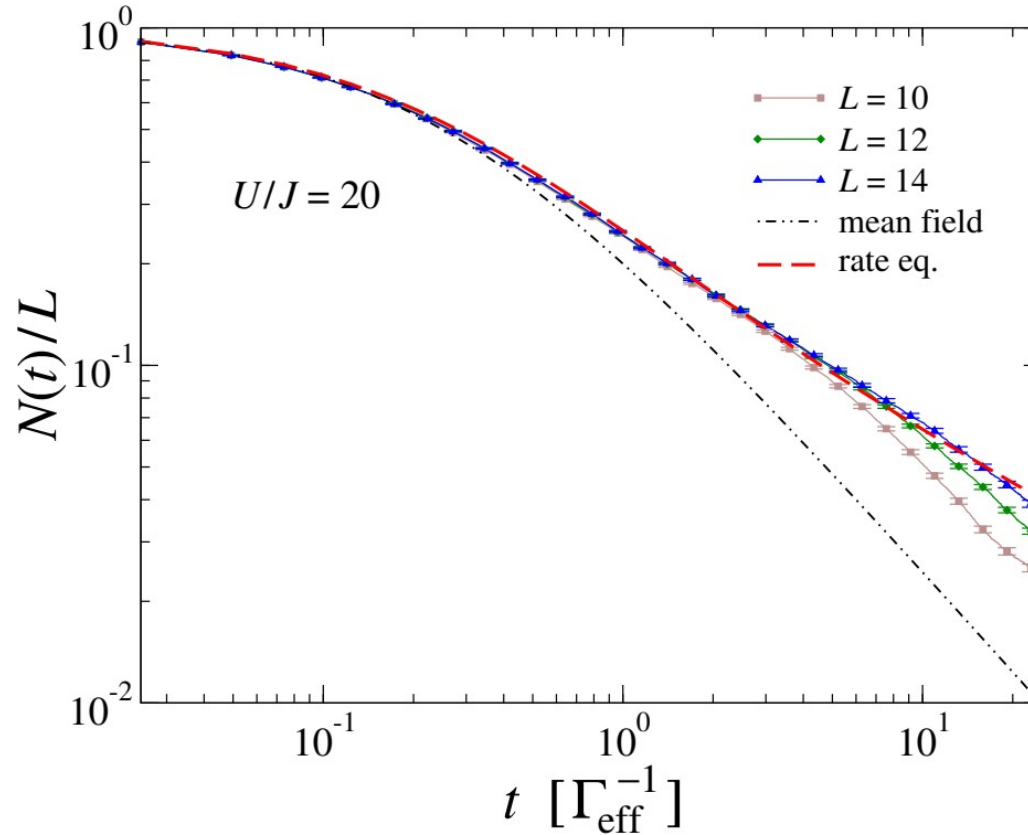
$\frac{1}{\Gamma_{\text{eff}}}$  Effective decay time for hard-core bosons




# An anomalous transient dynamics

Initial state:  
one particle per site

Exact simulations:  
Quantum-trajectory  
stochastic unraveling of the  
many-body master  
equation



We developed a simple analytical theory that describes the dynamics accurately. Here we plot its numerical solution.

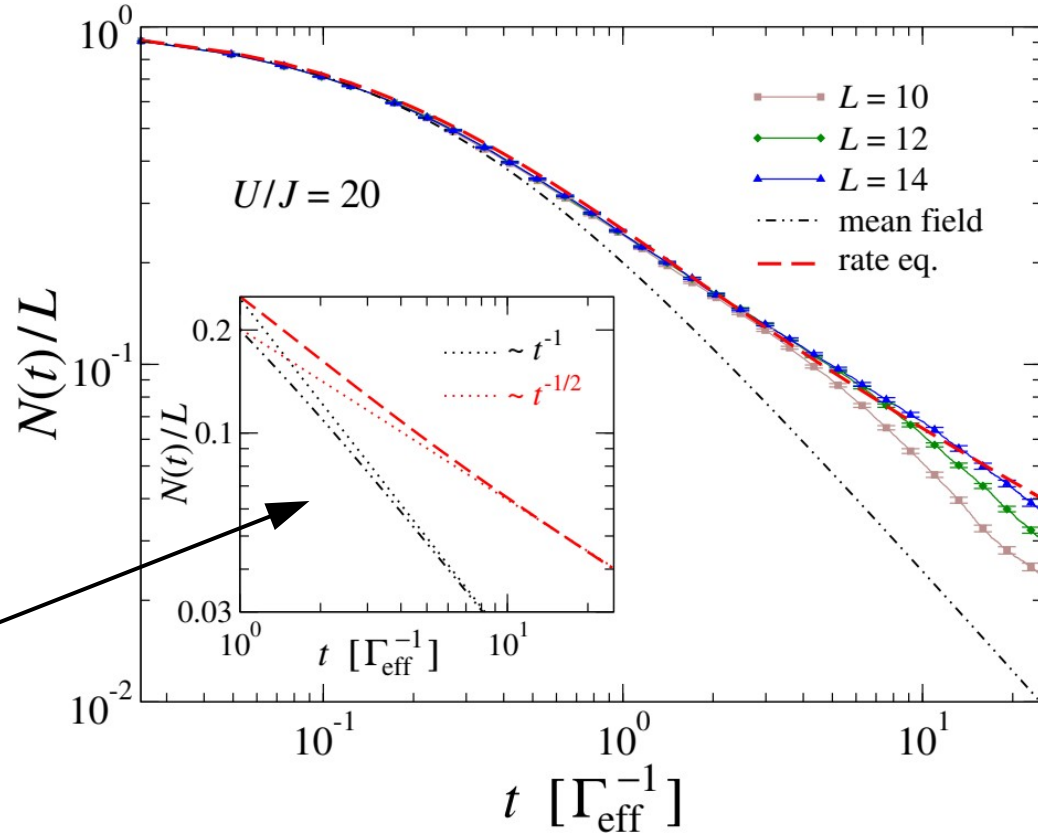

$$n(t) = \frac{1}{1 + \Gamma_{\text{eff}} t}$$

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Quantum and  
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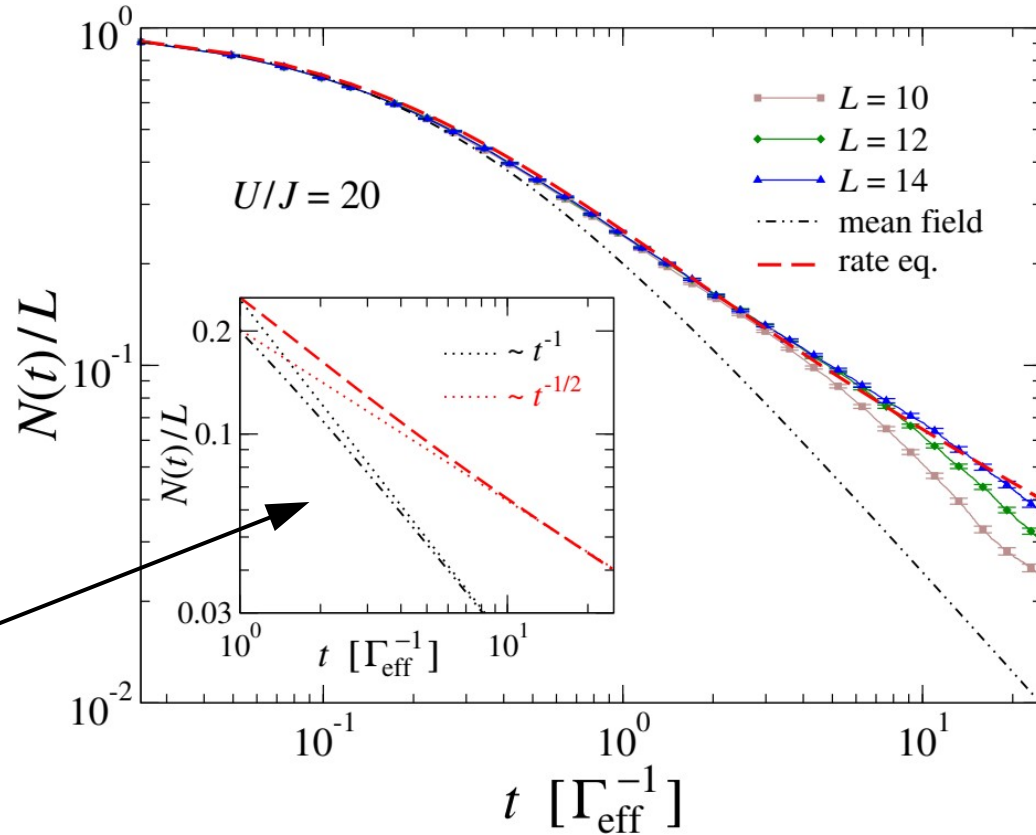
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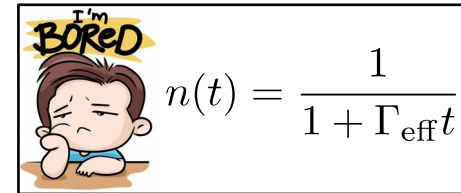
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We also have an approximate  
analytical formula

$$n(t) = \frac{\sqrt{1 + \frac{\Gamma_{\text{eff}} t}{2\pi}}}{1 + \Gamma_{\text{eff}} t}$$

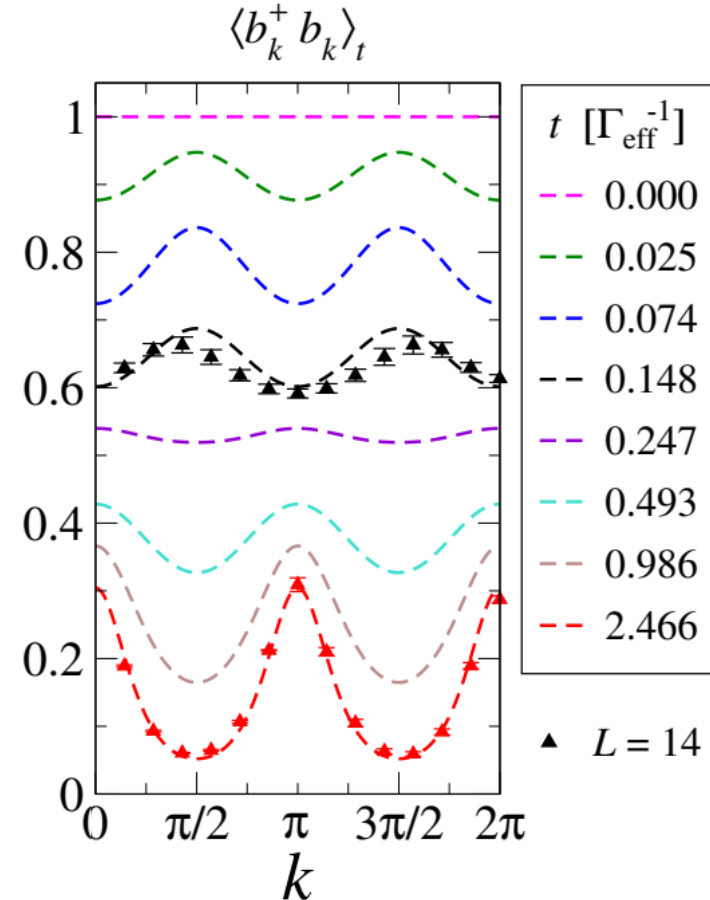
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# Momentum distribution function

## Bosonic momentum distribution function

- Clear peaks develop at different momenta at different times
- Observable in time-of-flight experiment
- No relation with an equilibrium distribution



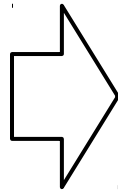
# Zero-range correlation function

The time-evolution of the population gives direct access to the correlation function of the gas

$$\frac{dn(t)}{dt} = -\Gamma_{\text{eff}} \langle \hat{n}(\hat{n} - 1) \rangle_t = -\Gamma_{\text{eff}} g_t^{(2)} n^2(t) \quad g_t^{(2)} = \frac{\langle b^\dagger b^\dagger b b \rangle_t}{\langle \hat{n} \rangle_t^2}$$

From the previous slide, we know that:

$$n(t) \sim \frac{1}{\sqrt{8\pi\Gamma_{\text{eff}}t}}$$



**Our result:** In the *long-time limit* we have an effective **three-body** interaction

$$\frac{dn(t)}{dt} = -4\pi\Gamma_{\text{eff}} n^3(t) \quad g_t^{(2)} \sim 4\pi n(t)$$

# How do we get these results?

## Time-scales of the problem

$$\frac{1}{\gamma}$$

Decay time of double occupancies or more

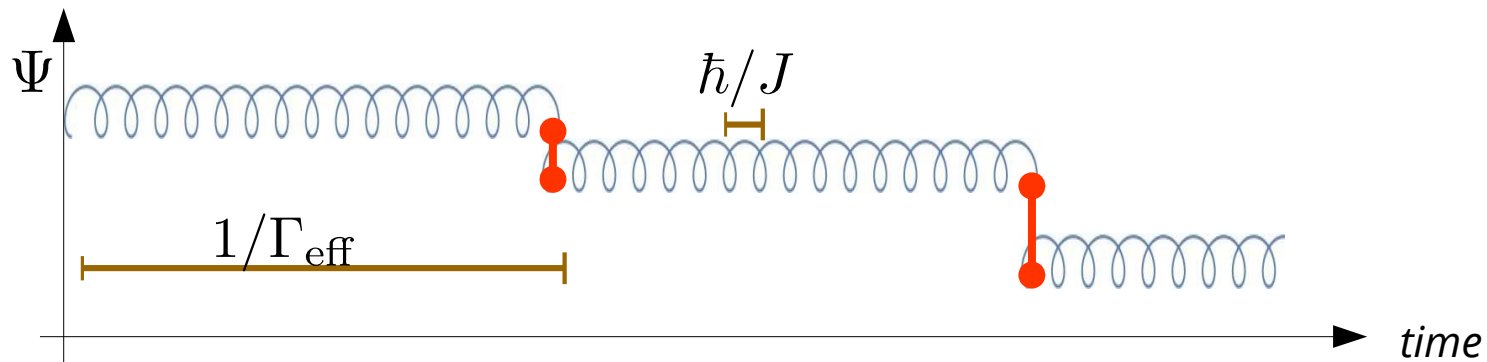
$$\frac{\hbar}{J}$$

Typical time of the hopping dynamics

$$\frac{1}{\Gamma_{\text{eff}}}$$

Decay time of single occupied sites

$$\dot{\rho} = -\frac{i}{\hbar} [H_1, \rho] + \Gamma_{\text{eff}} \sum_j \left[ c_j \rho c_j^\dagger - \frac{1}{2} (c_j^\dagger c_j \rho + \rho c_j^\dagger c_j) \right]$$



Intuitive picture: **loss events are rare.**  
Between two lossy events the system has a lot of time to evolve unitarily

# Generalised thermalisation

How to characterize the state

$$|\Psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}_1 t}|\Psi(0)\rangle$$

at long times?

$$H_1 = -J \sum_j \sigma_j^+ \sigma_{j+1}^- + \sigma_{j+1}^+ \sigma_j^-$$

Jordan-Wigner transformation

$$H_1 = -2J \sum_k \cos(k) c_k^\dagger c_k$$

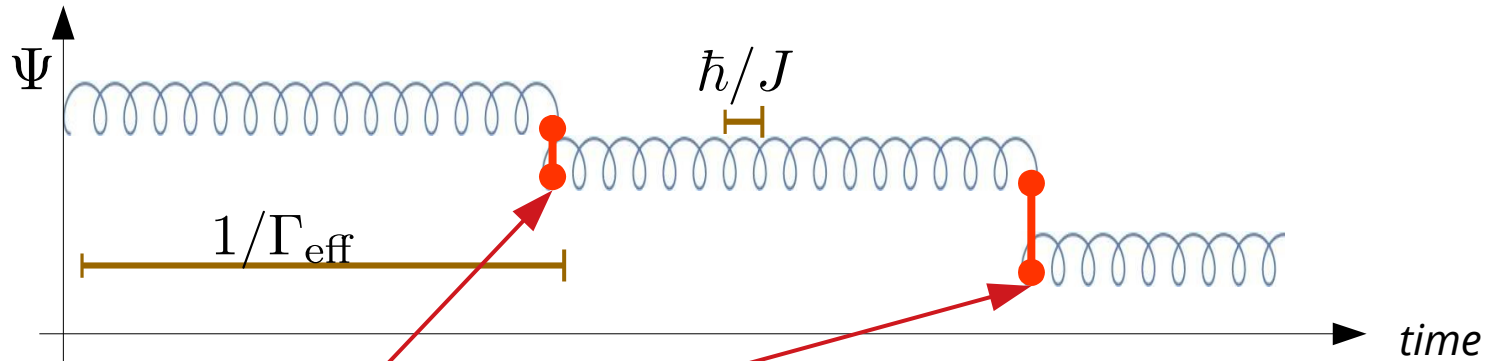
## Thermalization in closed quantum systems:

At long times the quantum state is indistinguishable from a  
generalised Gibbs state

$$\lim_{t \rightarrow \infty} \langle \Psi(t) | \hat{A}_j | \Psi(t) \rangle = \text{tr} \left[ \frac{e^{-\sum_k \lambda_k c_k^\dagger c_k}}{Z} \hat{A}_j \right]$$



# Time-dependent generalised thermalisation



At the moment of the loss event I only need to know the values of the appropriate Lagrange multipliers!

$$\rho(t) = \frac{e^{-\sum_k \lambda_k(t) c_k^\dagger c_k}}{Z}$$

Time-dependent generalised Gibbs ensemble

Lange, Lenarcic, Rosch PRB 2018



Write the equations satisfied by the generalised Lagrange multipliers



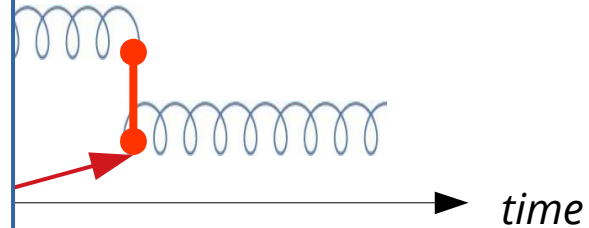
# Time-dependent generalised thermalisation

In practice:

- The time-dependent generalised Gibbs ensemble is a fermionic Gaussian state
- Compute the time-evolution of the occupation of each mode  $k$  with the master equation

$$\frac{d}{dt}n_k(t) = -\frac{4\Gamma_{\text{eff}}}{L} \sum_q n_q(t)n_k(t)[\sin(k) - \sin(q)]^2$$

Simple set of coupled first-order differential equations.  
Polynomial effort in the system size  $L$ .



$$\rho(t) = \frac{e^{-\sum_k \lambda_k(t) c_k^\dagger c_k}}{Z}$$

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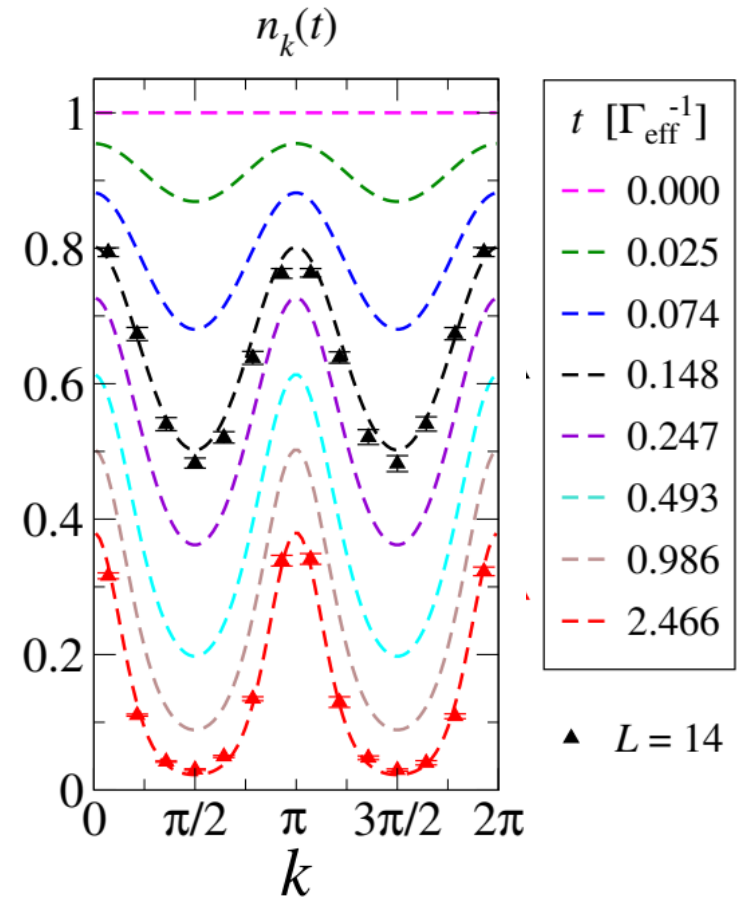


Write the equations satisfied by the generalised Lagrange multipliers

# Fermionic rapidities

- Observable with an expansion in the 1D tube (stable modes of the 1D dynamics)
- Clear peaks at well-defined quasimomenta

$$n_k(t) \sim f(t)e^{-g(t) \sin^2 k}$$

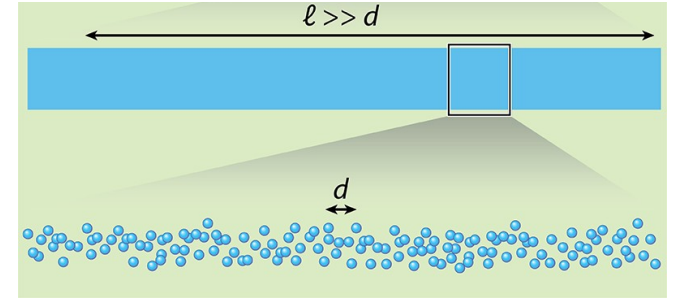


# A hydrodynamic theory in the trap

The rate equations are amenable to a treatment of the problem within the framework of the local density approximation.

- Two-body losses with a harmonic confinement.

$$n_k(t) \quad \rightarrow \quad n_k(x, t)$$



A Boltzmann-like equation

$$\frac{\partial n_k(x, t)}{\partial t} + \frac{\hbar k}{m} \frac{\partial n_k(x, t)}{\partial x} + F(x) \frac{\partial f_k(x, t)}{\partial k} = -\Gamma_{\text{eff}} \int (k - q)^2 f_k(x, t) f_q(x, t) dq$$

Harmonic confinement

$$F(x) = -\kappa x$$

# A hydrodynamic theory in the trap

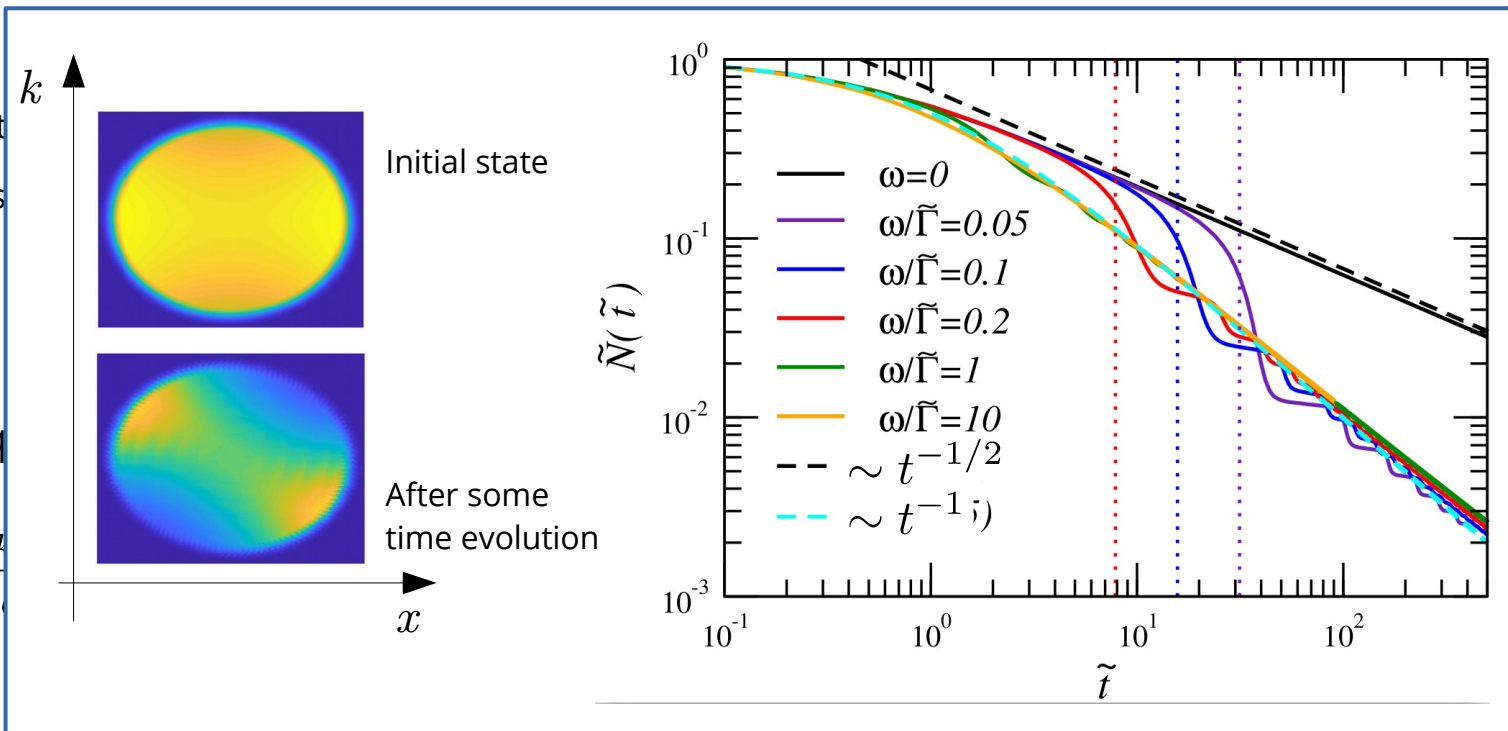
The rate equations  
the problem within  
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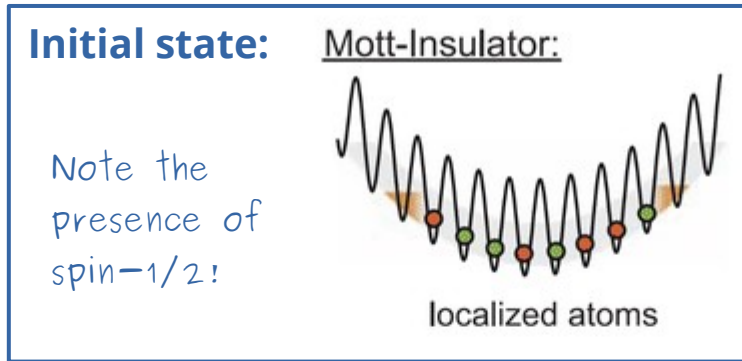
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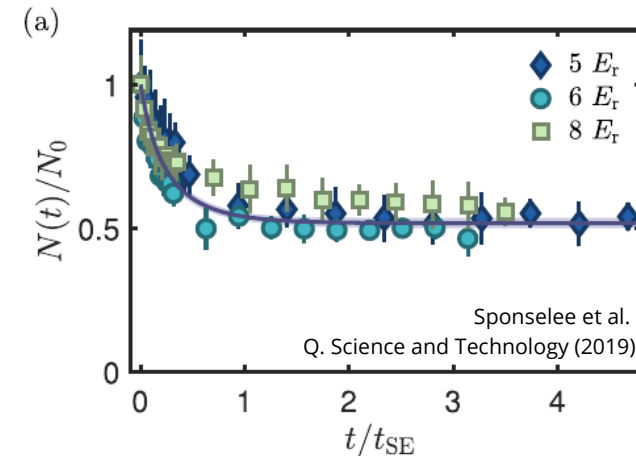
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- **Conclusions**

# Two-body losses in 1D spin $\frac{1}{2}$ fermi gases



## Quantum quench:

- Lower the optical lattice
- Atoms delocalise
- Two-body losses take place



**Experiments:** molecules (JILA) and ytterbium (Hamburg and Kyoto)

- The gas does not depopulate even if it is spin balanced.
- There is a finite population in the stationary state.

## Main previous theory works:

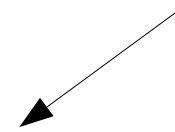
- Foss-Feig, Thomson, Daley, Rey, PRL 109, 230501 (2012)
- Nakagawa, Tsuji, Kawakami, Ueda, PRL 124, 147203 (2020)
- Nakagawa, Kawakami, Ueda, PRL 126, 110404 (2021)

# A master equation for 2-body losses

$$\dot{\rho} = -\frac{i}{\hbar} \left[ -J \sum_{j,\sigma} \left( c_{j,\sigma}^\dagger c_{j+1,\sigma} + H.c. \right) + U \sum_j n_{j,\uparrow} n_{j,\downarrow}, \rho \right] + \gamma \sum_j L_j \rho L_j^\dagger - \frac{1}{2} \{ L_j^\dagger L_j, \rho \}$$

The jump operator takes out one spin singlet  
→ **The dynamics conserves spin**  
→ The gas cannot depopulate completely

$$L_j = c_{j,\uparrow} c_{j,\downarrow}$$



# A master equation for 2-body losses

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**Example for 2 fermions.** Consider the spin part of the wavefunction:

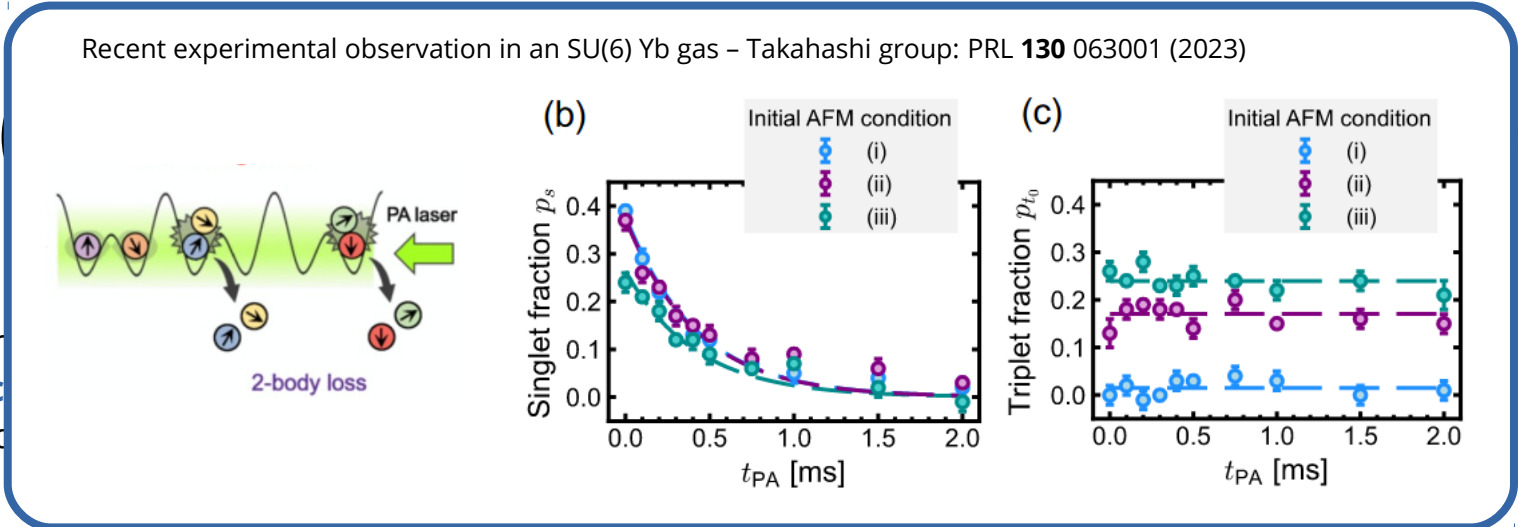
$ \uparrow\uparrow\rangle$		No particle losses
$ \downarrow\downarrow\rangle$		No particle losses
$\frac{1}{\sqrt{2}}( \uparrow\downarrow\rangle +  \downarrow\uparrow\rangle)$		No particle losses
$\frac{1}{\sqrt{2}}( \uparrow\downarrow\rangle -  \downarrow\uparrow\rangle)$		Particle losses are possible



# A master equation for 2-body losses

$$\dot{\rho} = -\frac{i}{\hbar} \left[ -J \sum_{j,\sigma} \right]$$

The jump operator  
 → The dynamics of the system  
 → The gas cannot be described by a Schrödinger equation



**Example for 2 fermions.** Consider the spin part of the waverfunction:

- $|\uparrow\uparrow\rangle$  → No particle losses
- $|\downarrow\downarrow\rangle$  → No particle losses
- $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$  → No particle losses
- $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$  → Particle losses are possible

# Dynamical theories for fermionic gases

## Weakly interacting and weakly dissipative regime

Our result: A correction to the mean-field equation that takes into account the spin conservation in the thermodynamic limit

$$\dot{n}(t) = -\kappa \left( n(t)^2 - \frac{4}{\hbar^2} \frac{\langle \hat{S}^2 \rangle}{L^2} \right) \xrightarrow{\text{stationary state}} \langle S^2 \rangle \approx \hbar^2 \left( \frac{N}{2} \right)^2 \approx \hbar^2 \frac{N}{2} \left( \frac{N}{2} - 1 \right)$$

In the thermodynamic limit, the stationary state has maximal spin  
A Dicke state! **Entanglement through dissipation!**

Rosso, Biella, Rossini, LM PRA **104**, 053305 (2021)

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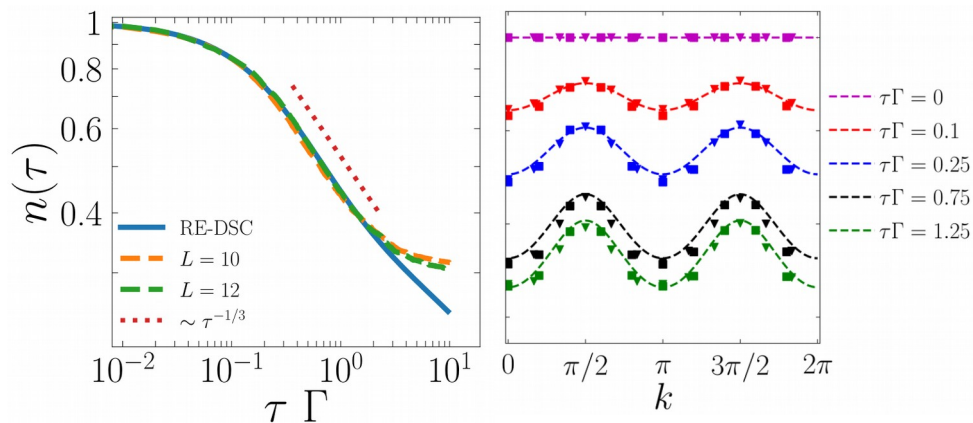
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Rosso, Biella, Rossini, LM PRA **104**, 053305 (2021)

## Strongly interacting or strongly dissipative regime

Our result: Study of the quantum Zeno regime. A modified theory for fermionised fermions.



Rosso, Biella, De Nardis, LM PRA **107** 013303 (2023)

# Conclusions

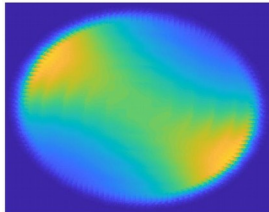
The lossy dynamics of one-dimensional quantum gases results from the interplay of unitary and dissipative dynamics:

Entangled stationary states & non-equilibrium dynamics

**Our contribution:** we have developed a theory for the dynamics of 1D lattice gases in the weakly- and strongly-dissipative regimes, focusing on two-body losses

## Bosonic gas

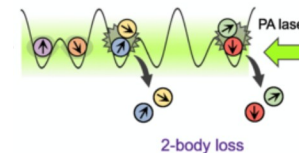
- Rate equations for the dynamics of the rapidities
- Hydrodynamical theory of the trap



D. Rossini, ... J. Beugnon, F. Gerbier, LM, PRA **103** L060201 (2021)  
Rosso, Biella, LM, SciPost Phys. **12**, 044 (2022)

## Fermionic gas

- Spin conservation and entangled steady states
- 2-body losses in  $SU(N)$  gases
- Quantum Zeno effect and dissipative spin cooling



Rosso, Biella, Rossini, LM PRA **104**, 053305 (2021)  
Rosso, Biella, LM PRA **105**, L051302 (2022)  
Rosso, Biella, De Nardis, LM PRA **107** 013303 (2023)

# Perspectives - experiments

A generation of **old-style experiments**

- performed when theory was not sufficiently developed
- Average over many 1D tubes hides interesting non-equilibrium scalings

**Main result:** No probe of emerging correlations, just the emergence of a new lifetime

# Perspectives - experiments

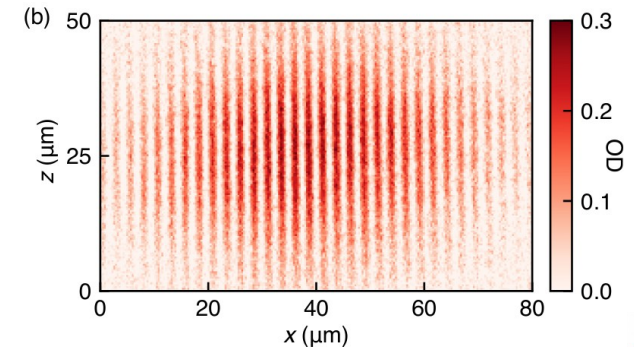
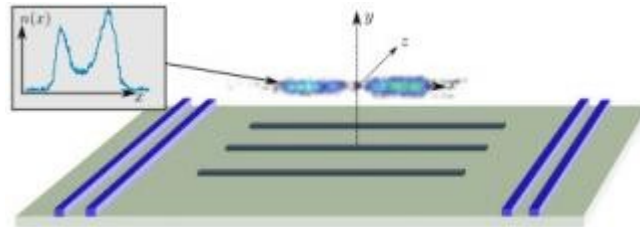
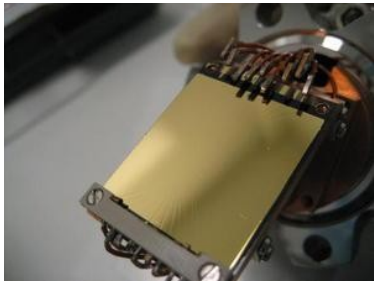
A generation of **old-style experiments**

- performed when theory was not sufficiently developed
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**Main result:** No probe of emerging correlations, just the emergence of a new lifetime

## New activities

- Takahashi: small two-sites double-wells. First clean data on the generation of entanglement.
  - Can we scale it up?
- One-dimensional resolved experiments (quantum gas microscope, atom chip experiments...)



# Acknowledgements



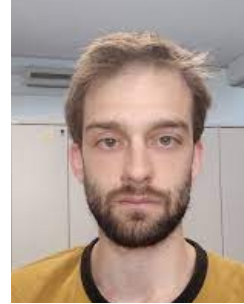
Guillaume Roux



Lorenzo Gotta  
LPTMS → Geneva



Lorenzo Rosso



Alberto Nardin  
Trento → LPTMS

The group on  
1D quantum  
matter @ LPTMS

Our recent  
collaborators  
on these  
projects



Jacopo De Nardis



Alberto Biella  
LPTMS → Trento

# Thank you

The new LPTMS  
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