Robust universal quantum processors in spin systems via Walsh pulse sequences (arxiv:2311.10600)

Joint work with M. Votto (LPMMC) and J. Zeiher (MPQ)

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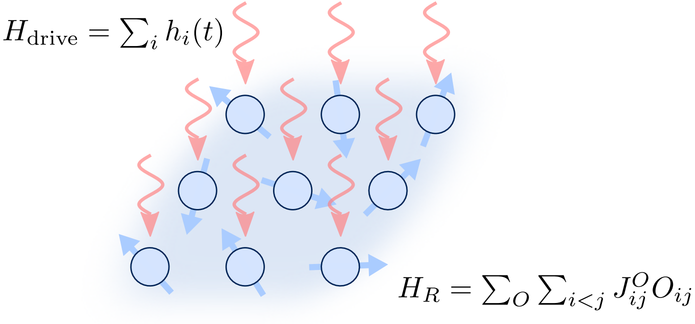
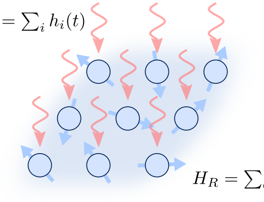
Université Grenoble-Alpes, LPMMC

Programmable quantum simulators



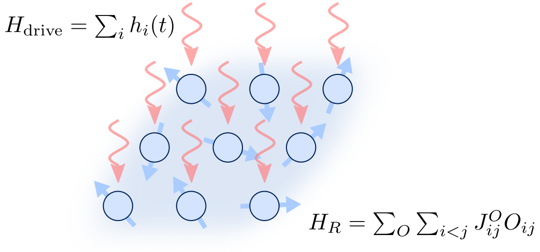
What kind of (robust) dynamics $|\psi(t)\rangle = U(t)\,|\psi(0)\rangle$ can be accessed with such a setup?

Programmable quantum simulators



- **Theorem**: any non-trivial 2-local spin Hamiltonian H_R with local pulses is sufficient for **universal quantum computation** *(Dodd et al., PRA 2002)*
- This inspired the development of **pulse sequences-based programmable quantum simulation** schemes:
  1. Engineering of translation-invariant Hamiltonians *(Hayes et al., NJP 2014)*
  2. Digital-analog quantum computation *(Parra-Rodriguez et al., PRA 2020)*
  3. Robust experimental realizations of *XYZ* models *(Choi et al. PRX 2020, ...)*

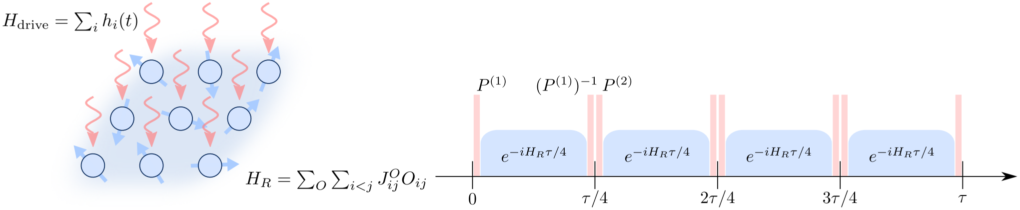
In this talk



We formulate a uniyfing approach which

- can engineer **arbitrary** 2-local Hamiltonian dynamics $U(t) = e^{-i\tilde{H}t}$ in an **efficient** way
- is **robust** to experimental imperfections by design
- is implementable in state-of-art setups (e.g. trapped ions, Rydberg atom arrays)

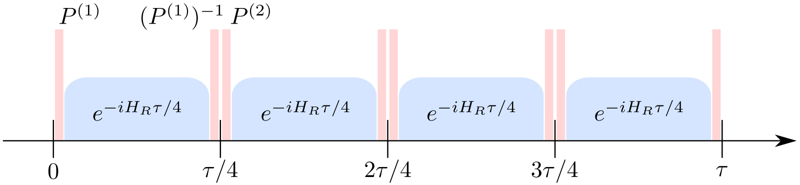
Hamiltonian engineering



- Assumption (temporary): dynamics of the external fields is **fast enough** and **isolated in time**
- We have the **pulse sequence** $\{P^{(k)} = \bigotimes_i p_i^{(k)}\}_{k=1}^n$ over a **period** τ with $p_i = e^{-it_p h_i}$
- For concreteness, we consider the resource Hamiltonian (c.f Thierry's/Clément's talk)

$$H_R = \sum_{i>j} (J_{ij}^X X_i X_j + J_{ij}^Y Y_i Y_j)$$

Hamiltonian engineering

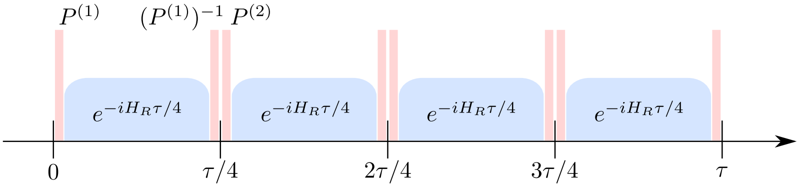


- Resulting dynamics: $\psi(\tau) = \tilde{U}(\tau)\psi(0)$

$$\tilde{U}(\tau) = \prod_{k=1}^{n} (P^{(k)})^{-1} e^{-iH_R\tau/n} P^{(k)} = \prod_{k=1}^{n} e^{-iH^{(k)}\tau/n}$$

- $H^{(k)} := (P^{(k)})^{-1} H_R P^{(k)}$ are called **toggling-frame Hamiltonians**

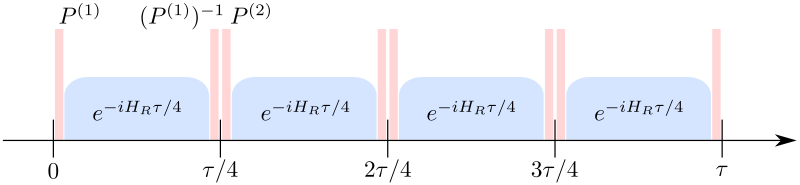
Hamiltonian engineering



- Goal of Hamiltonian engineering: How to program $\{P^{(k)}\}$ to realize a target Hamiltonian dynamics $\tilde{H}$ over a time T?
- Mathematically, we would like to have

$$[\tilde{U}(\tau)]^{T/\tau} \approx e^{-i\tilde{H}T}$$

Engineering via Average Hamiltonian Theory

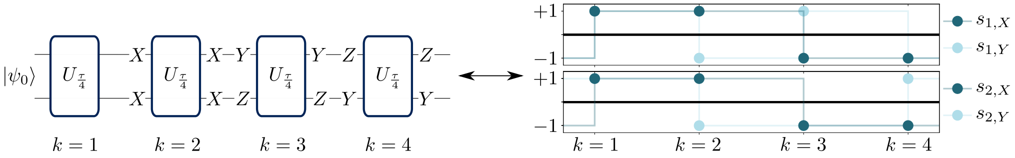


- **Average Hamiltonian Theory** (AHT, *Kuwahara et al., AoP 2016*):

$$(\tilde{U}(\tau))^{T/\tau} = \left(\prod_{k=1}^{n} e^{-iH^{(k)}\tau/n}\right)^{T/\tau} = e^{-i\tilde{H}T} + O(\tau T)$$

- $\tilde{H} = \frac{1}{n}\sum_k H^{(k)} = \frac{1}{n}\sum_k (P^{(k)})^{-1} H_R P^{(k)}$ is the **average Hamiltonian**
- How do I reverse engineer the pulses $P^{(k)}$ to form an arbitrary $\tilde{H}$?
- We shall use two reductions to solve this difficult problem

Reduction 1: From pulse matrices to sign coefficients

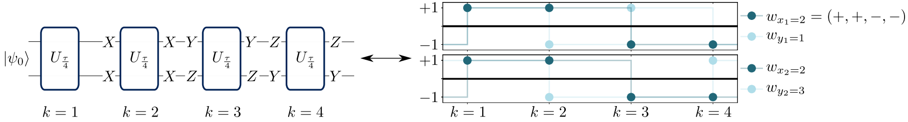


- Let us consider $\pi-$pulses: $p_i^{(k)} \in \{1, X, Y, Z\}$

$$H^{(k)} = (P^{(k)})^{-1} H_R P^{(k)} = \sum_{i>j} (s_{i,X}^{(k)} s_{j,X}^{(k)} J_{ij}^X X_i X_j + s_{i,Y}^{(k)} s_{j,Y}^{(k)} J_{ij}^Y Y_i Y_j)$$

  where $s_{i,O}^{(k)} = \pm 1$ (Pauli commutation relations, eg $ZXZ = -X$, etc)
- The functions $s_{i,O}^{(k)}$ can be chosen arbitrarily (4 different pulses $p_i^{(k)}$ for 2×2 different signs $s_{i,X,Y}^{(k)}$).

Reduction 2: Parameterize the signs with Walsh pulse sequences



- The average Hamiltonian is related to **scalar products** between sign functions

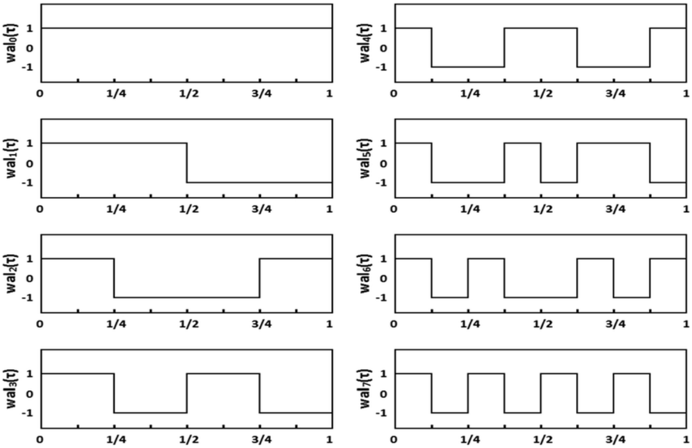
$$\tilde{H} = \frac{1}{n}\sum_k H^{(k)} = \sum_{i>j}\left((s_{i,X}|s_{j,X})J_{ij}^X X_i X_j + (s_{i,Y}|s_{j,Y})J_{ij}^Y Y_i Y_j\right)$$

$$(s_{i,O}|s_{j,O}) = \frac{1}{n}\sum_k s_{i,O}^{(k)} s_{j,O}^{(k)}$$

- We choose **Walsh functions** $s_{i,O}^{(k)} = w_a^{(k)}$, set of orthonormal sign functions *(Walsh, AJM (1923))*, $0 \le a \le n-1$

$$(w_a|w_b) = \delta_{a,b}$$

Example of Walsh functions with $n = 8$

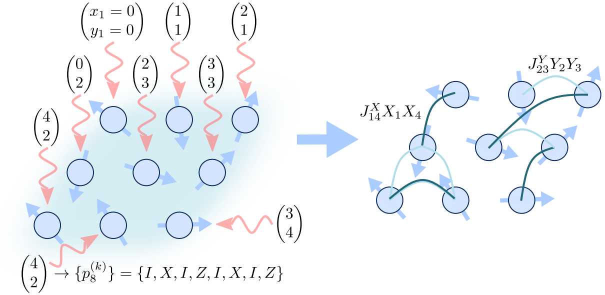


Source: Bajalinov & Duleba 10.1007/s10100-019-00614-3.

From Walsh pulse sequences to interaction graphs

- We assign two Walsh indices (x_i, y_i) per qubit

$$\tilde{H} = \sum_{i>j} (\delta_{x_i,x_j} J_{ij}^X X_i X_j + \delta_{y_i,y_j} J_{ij}^Y Y_i Y_j)$$

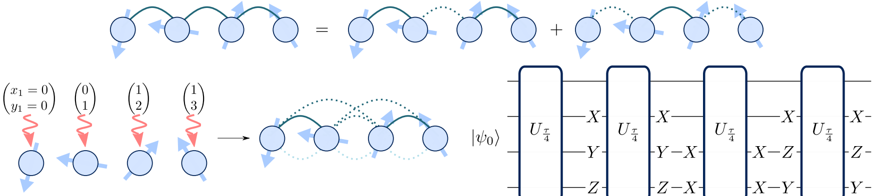


- We have represented $\tilde{H}$ in terms of **interaction graphs** G^X, G^Y.

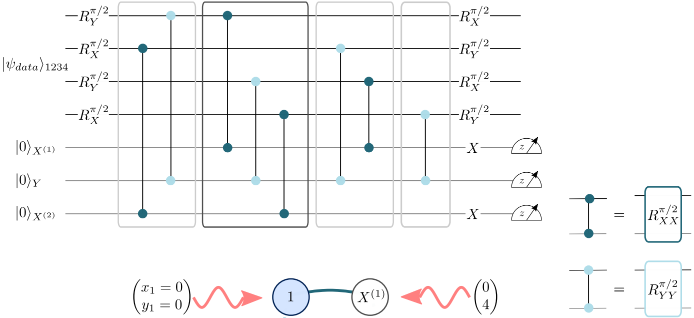
Hamiltonian engineering via Walsh pulse sequences

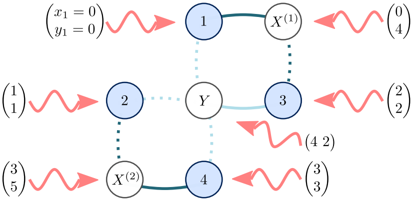
- **Theorem:** Any arbitrary 2-local Hamiltonian can be realized using Walsh pulse sequences (proof hint: graph decomposition)
- Example 1: from a long-range XX chain to the nearest-neighbor Ising chain

$$\text{Resource } J_{ij}^X = J_{ij}^Y = -\frac{J}{|i-j|^\alpha} \rightarrow \text{Target } \tilde{H} = \tilde{H}_1 + \tilde{H}_2 = -J\sum_i X_i X_{i+1}$$

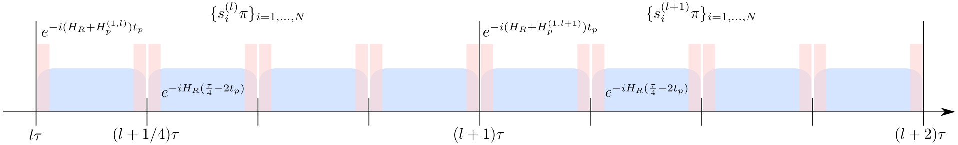


Example 2: Surface code from Walsh sequences



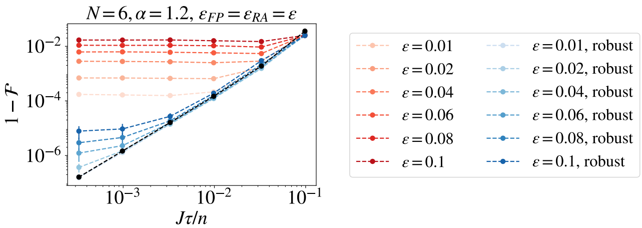


Robustness conditions from double averaging



- Choosing signs $s_i^{(l)} = w_{e_i}^{(l)}$ for generating the pulses
- **Robustness conditions** for finite pulse duration errors $e_i \neq e_j,\ e_i \neq 0 \to \Delta\tilde{H} = 0$

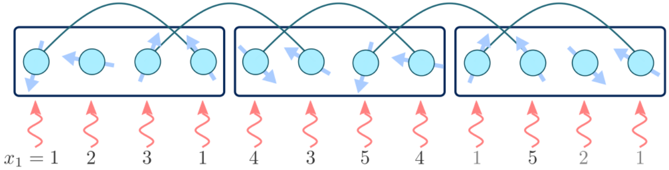
Numerical example: nearest-neighbor Ising chain



- We consider $\tilde{H} = H_{\text{Ising}},\ T = \pi/4$
- Rotation angle errors are introduce randomly as $(\pi + \delta_i)-$pulses, sampling $\delta_i \in [-2\epsilon_{\text{RA}}, 2\epsilon_{\text{RA}}]$ uniformly
- We observe that robustness conditions improve the fidelity by **orders of magnitude**

Summary of the technical details

- The protocol is robust in leading order in t_p/τ in finite pulse duration errors
- Robustness also with respect to static fields $\sum_i h_i Z_i$, and rotation angle errors.
- Theory relies on average Hamiltonian theory: Trotter errors can be analytically bounded as a function of $J\tau \ll 1$.
- Pulse frequency n/τ can be reduced by using restricted sets of Walsh functions



Summary

- A simple experimental recipe for implementing any circuit/Hamiltonian/Optimization problem using local pulses.
- Precise ressource analysis: number of pulses, frequency
- Analytical theory provides powerful robustness aspects.
- More details, numerical examples, etc, in arxiv:2311.10600