EVIDEN

Combinatorial optimization with Rydberg platforms: advances and challenges

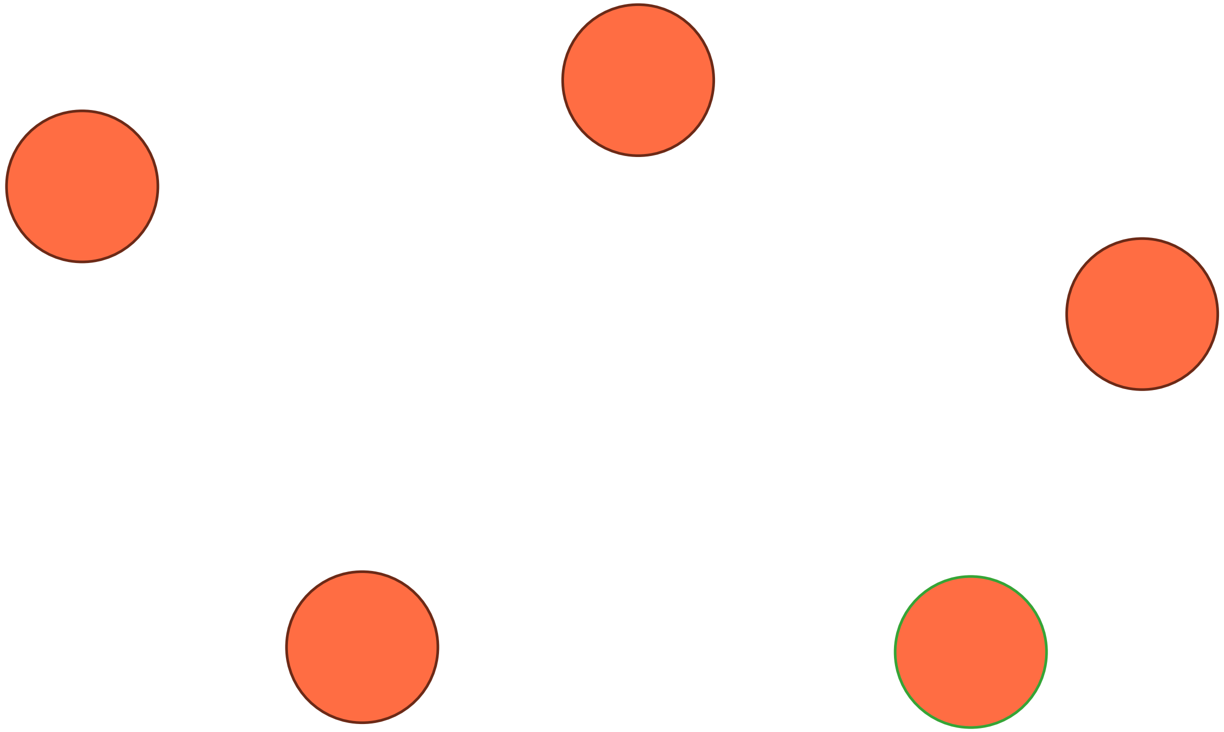
Colloque Atomes de Rydberg et simulation quantique
Friday, April 5th, 2024

Thomas Ayral
Eviden Quantum Lab, France

an atos business

An example of combinatorial optimization problem

A successful party



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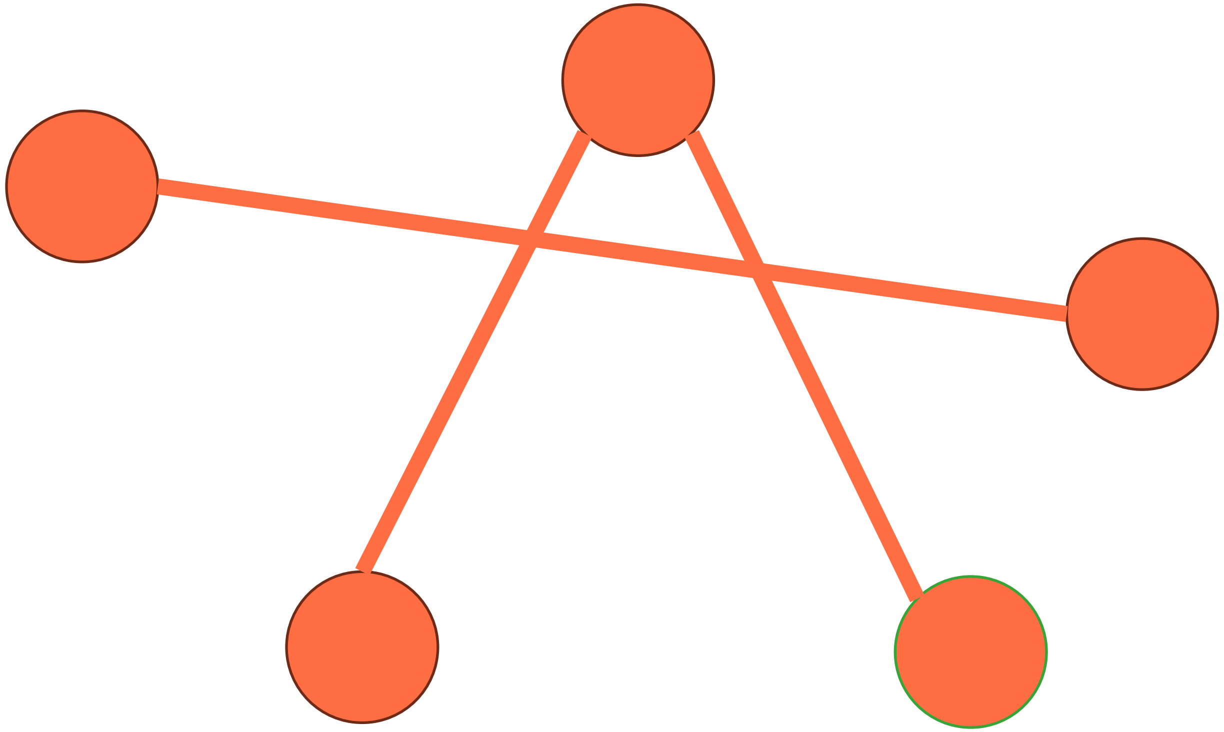

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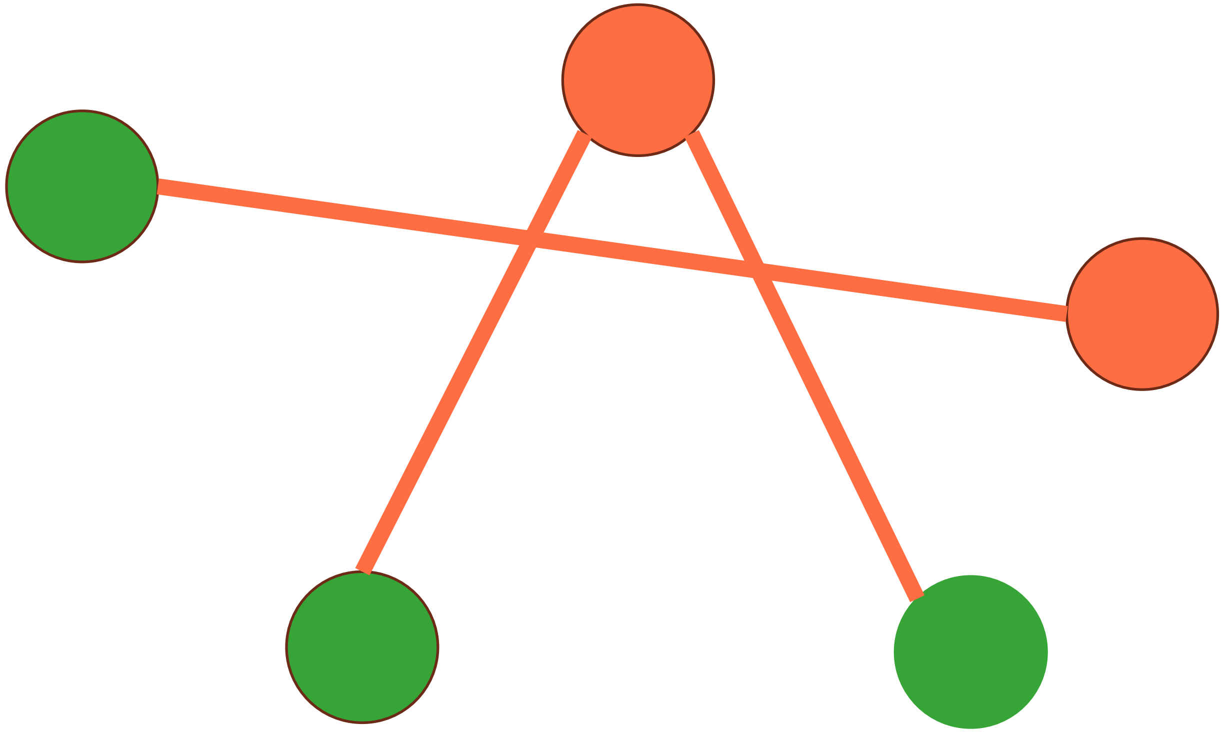

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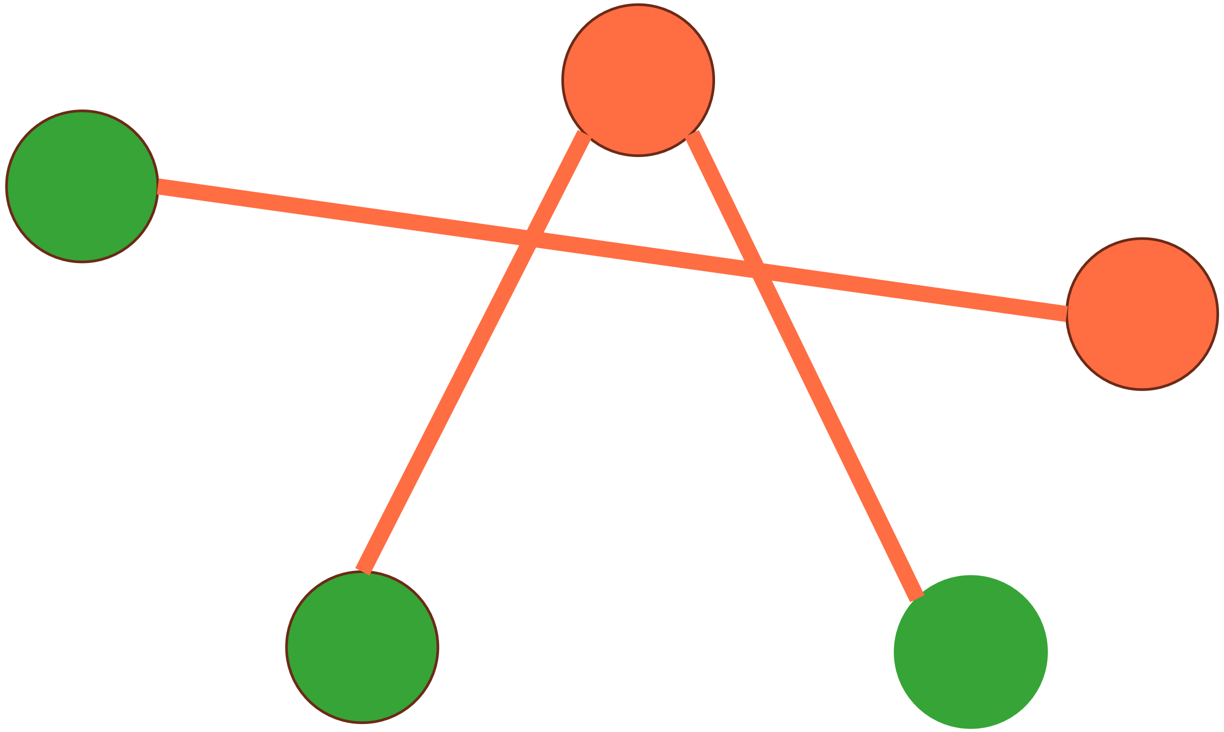
Maximum independent set (MIS) problem



Optimal solution?

runtime exponential with system size!

Approximate solution?

hard limit to get close to optimal solution!

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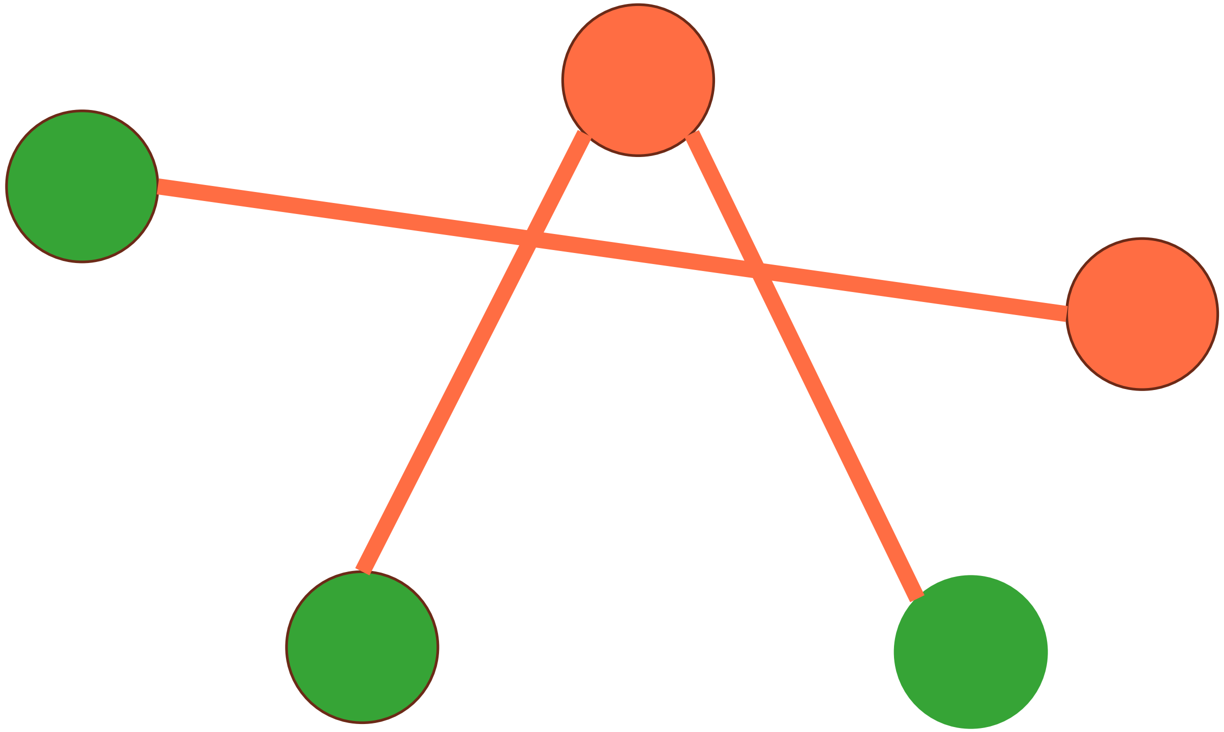

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Many other examples
Traveling salesperson, maximum cut problem...

Many industrial applications!

Can (Rydberg) quantum processors help?

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Outline

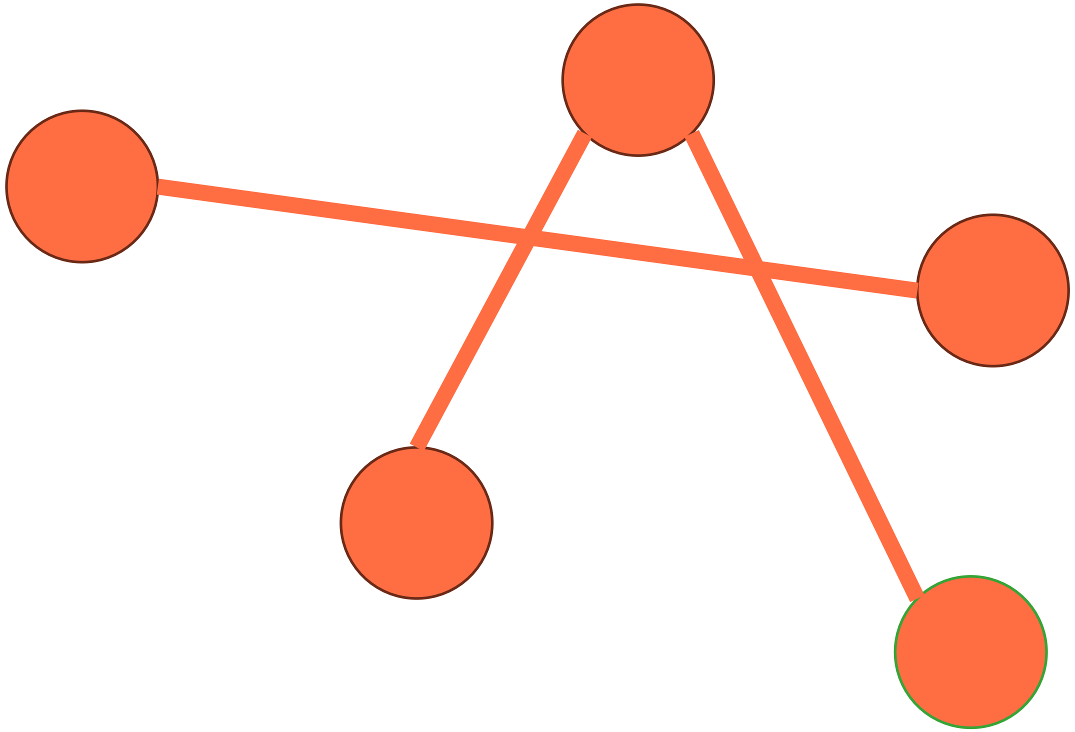
1
The unit-disk maximum independent set problem:
a promising application for Rydberg quantum simulators?

2
The challenge of decoherence

3
Towards more general graphs... and applications

1 The unit-disk maximum independent set problem: a promising application for Rydberg quantum simulators?

The unit-disk maximum independent set



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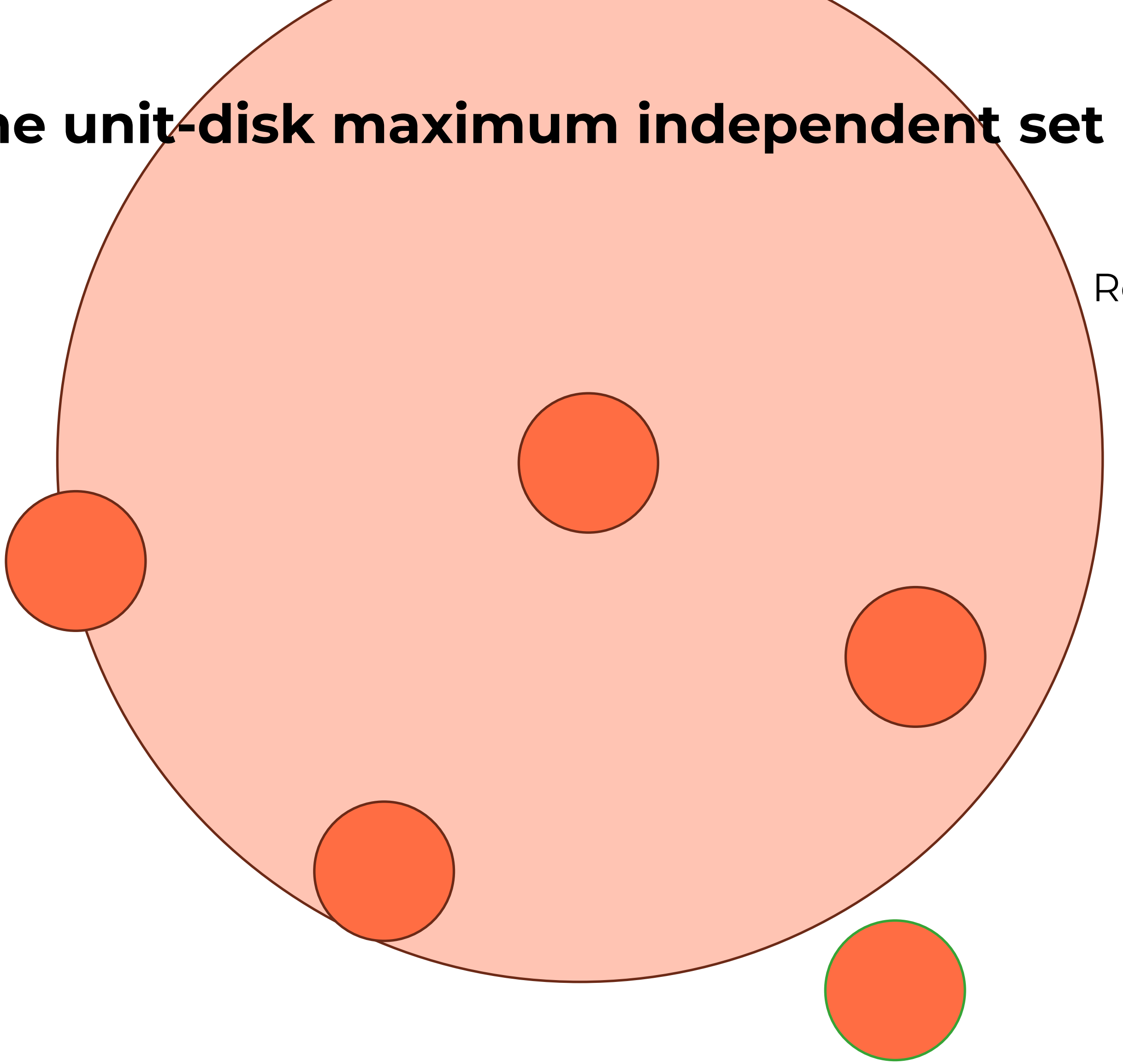

Restriction to unit-disk graphs: **UDMIS**

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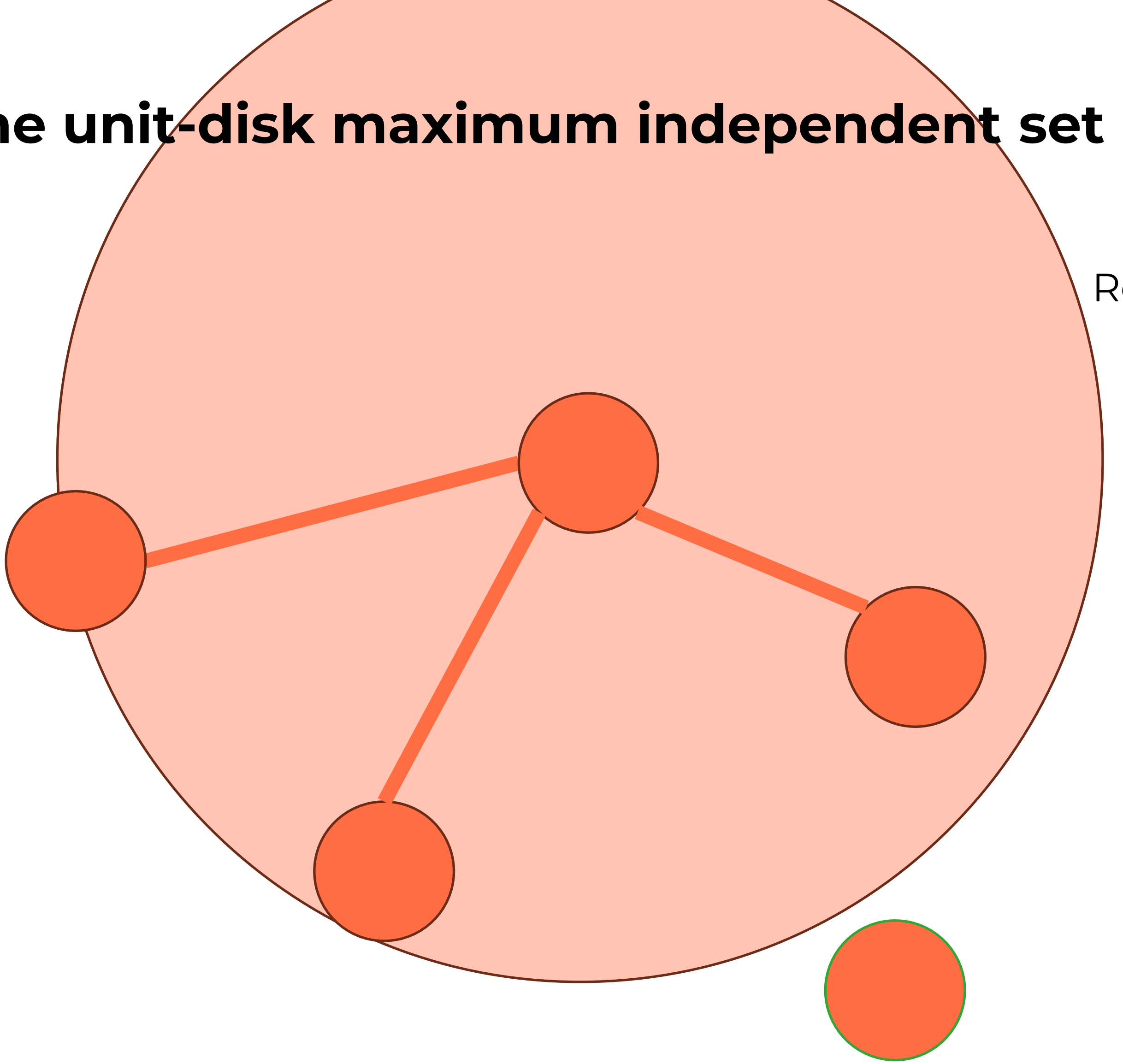

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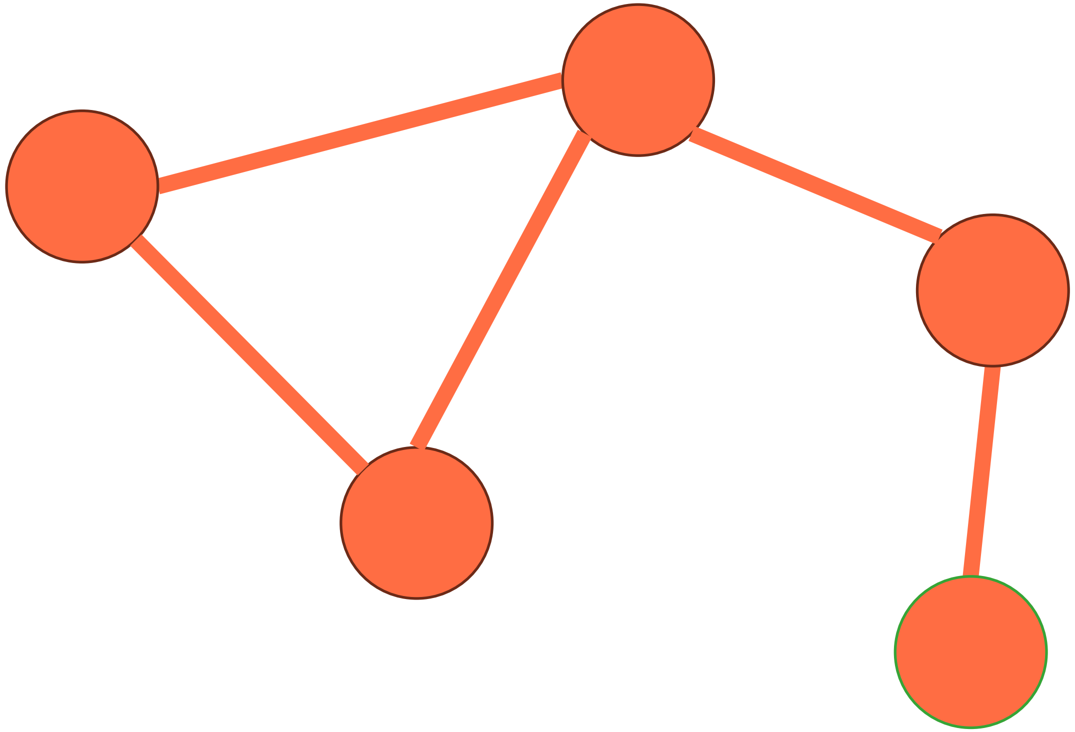

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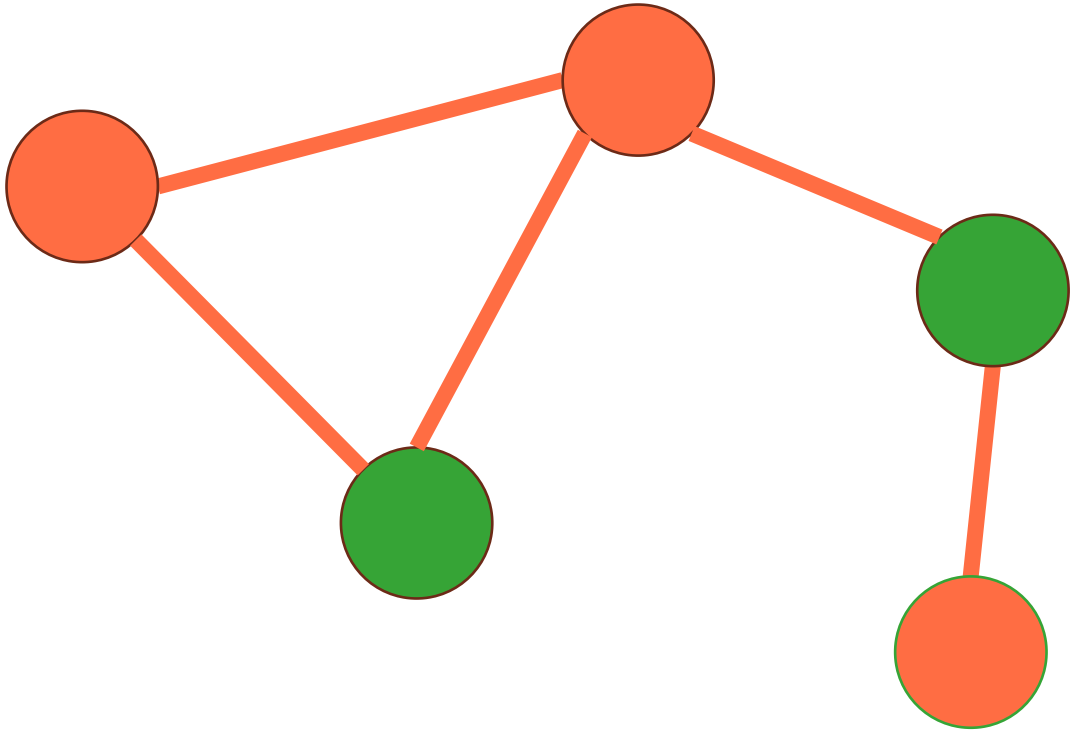

Restriction to unit-disk graphs: **UDMIS**

Still an exponential runtime for optimal solution.

For approximate solution:

can get ϵ close to optimal, but **runtime is exponential in $1/\epsilon$!**

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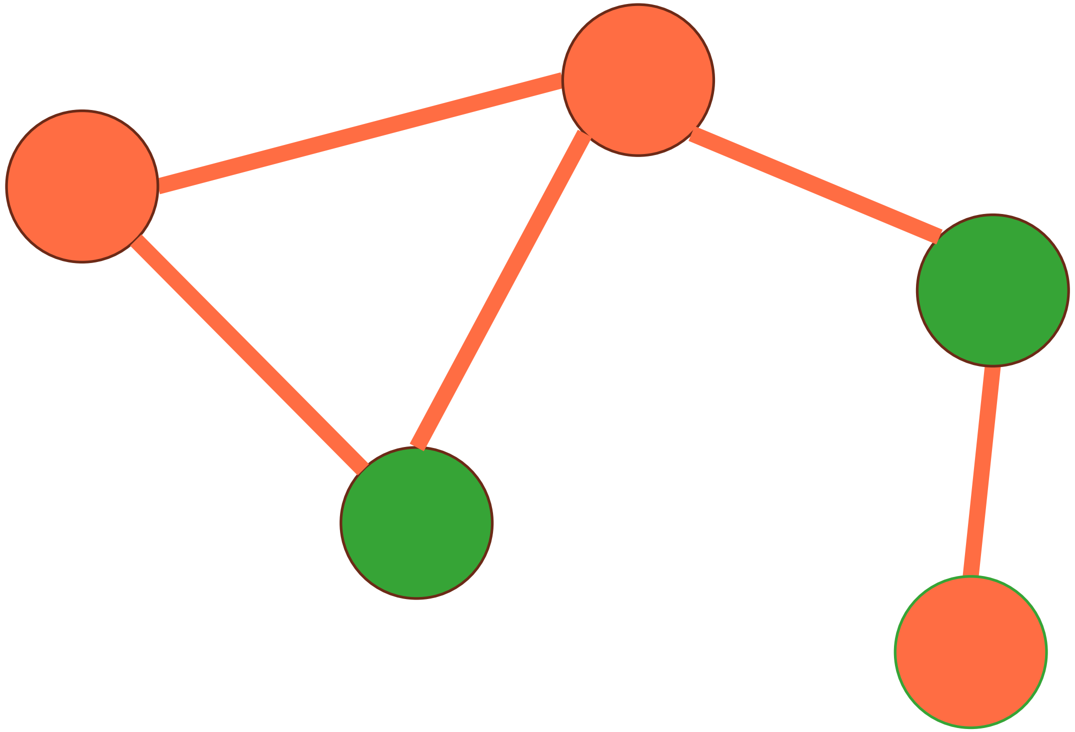

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Precise definition of success?

Approximation ratio:

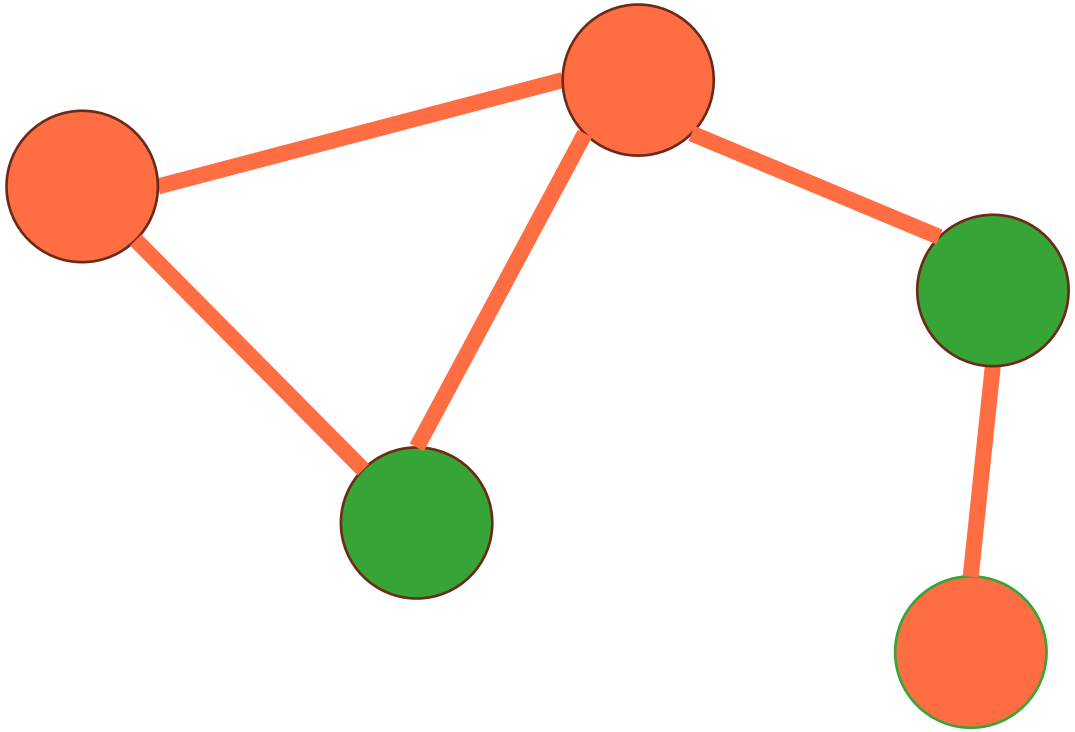
Your solution

Cost function

$$\alpha = \frac{C(S)}{C(S^*)} \leq 1$$

Optimal solution

The unit-disk maximum independent set



Restriction to unit-disk graphs: **UDMIS**

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For approximate solution:
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$$\alpha = 1 - \epsilon$$

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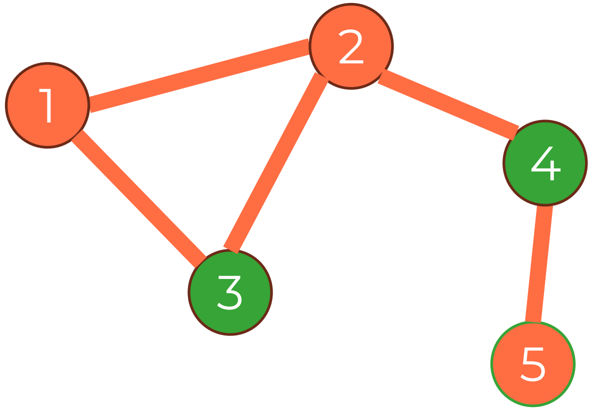
Optimal solution

How could quantum mechanics help?

Turn cost function minimization into estimation of ground state of Hamiltonian!

One solution = a string of bits $S = (n_1, n_2, \ldots, n_n)$

Here $S = (0,0,1,1,0)$



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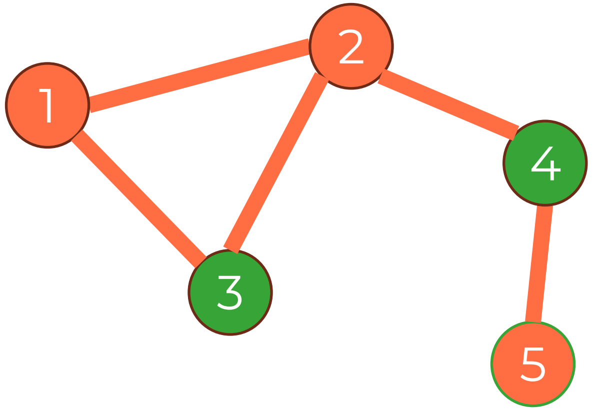

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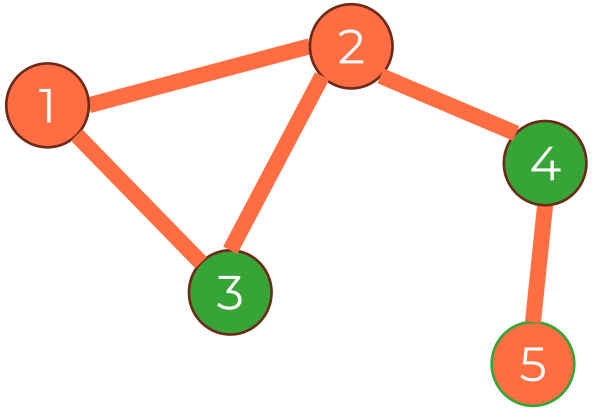

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$$C(S, U) = \sum_i n_i - U' \sum_{i,j \in E} n_i n_j$$

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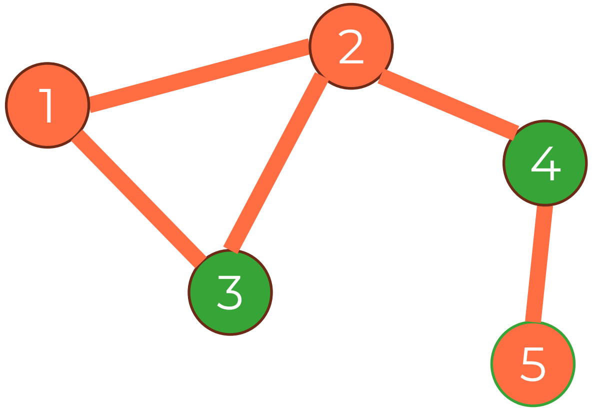

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Turn into operators (+minus sign):

$$H = -\delta \sum_i \hat{n}_i + U \sum_{i,j \in E} \hat{n}_i \hat{n}_j$$

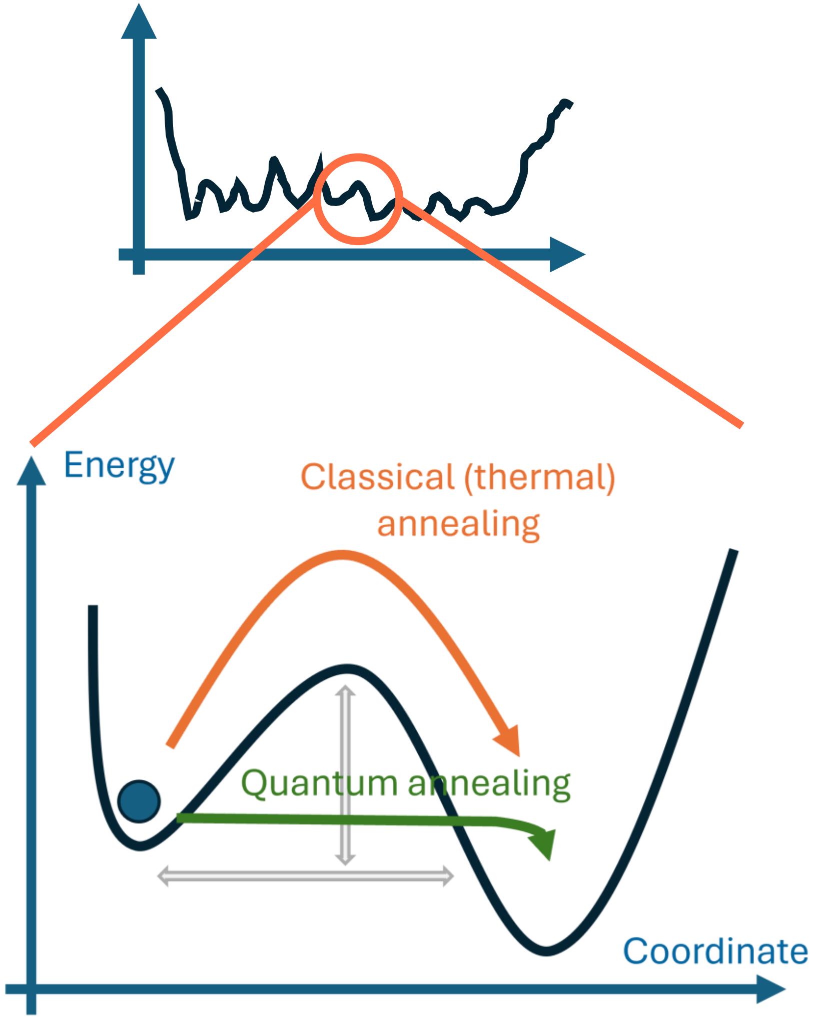
with $\hat{n}|0\rangle = 0, \hat{n}|1\rangle = |1\rangle$.

Equivalent quantum problem:

Find lowest eigenstate

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle$$

Finding a quantum ground state in practice: quantum annealing



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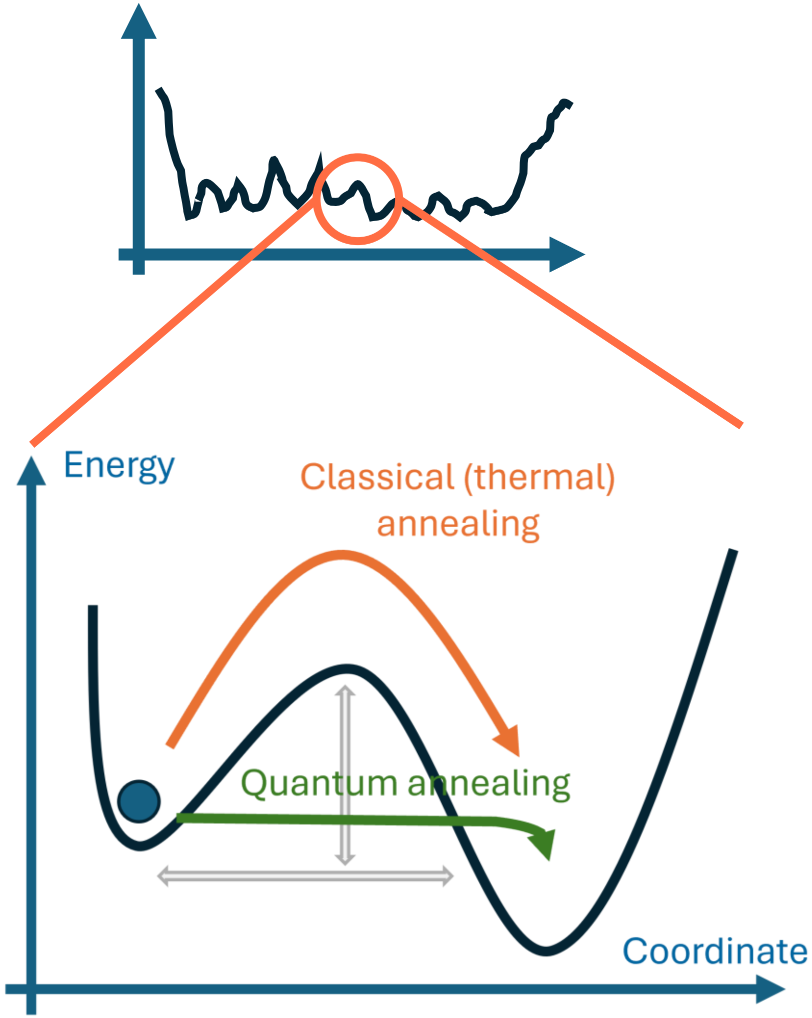


To create quantum tunneling:

$$H(t) = \frac{t}{t_f} H + \left(1 - \frac{t}{t_f}\right) H_{\text{tunnel}}$$

with e.g

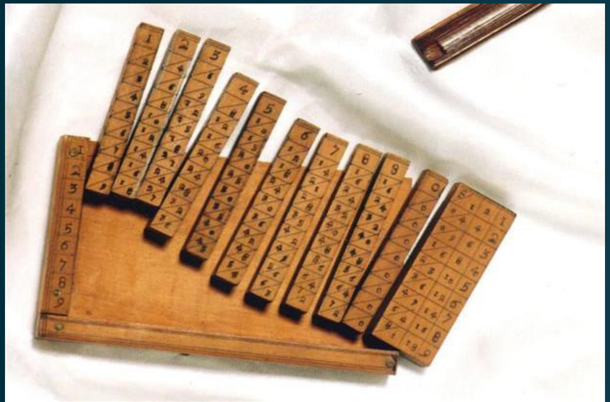
$$H_{\text{tunnel}} = \Omega \sum_i \hat{\sigma}_i^x$$

If annealing time t_f long enough (adiabatic):

Start from GS $|\Phi_0\rangle$ of H_{tunnel}, end in GS $|\Psi_0\rangle$ of H.

One experimental implementation: **d-wave computers: Classical + quantum annealing**.

Analog computers?



Slide rule
(J. Napier, 1614)

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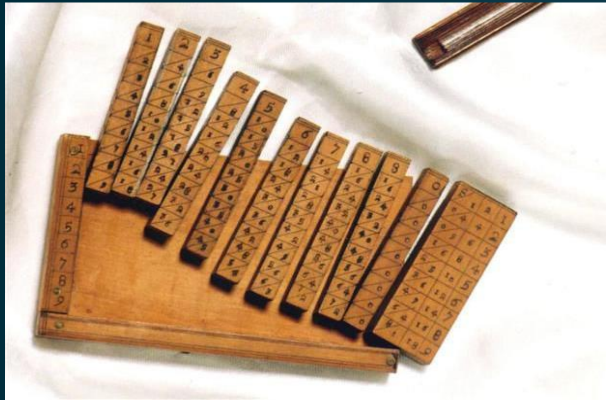

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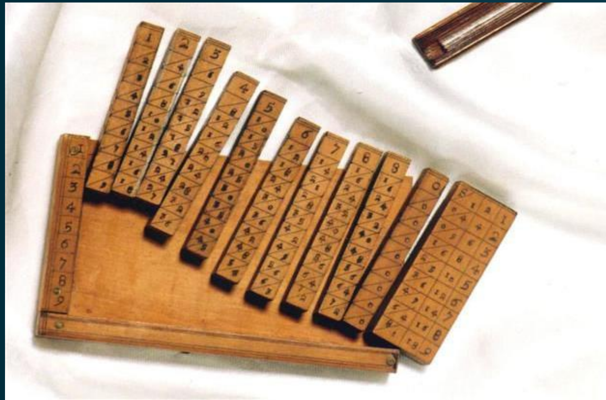

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Bull machine
(F. Bull, 1920s)

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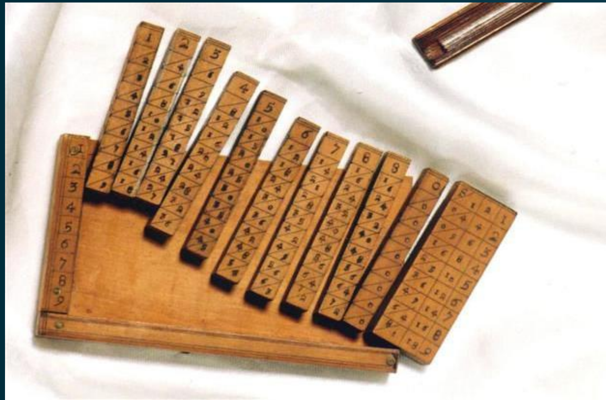

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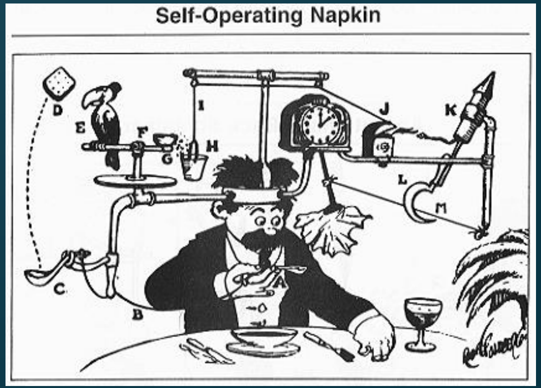


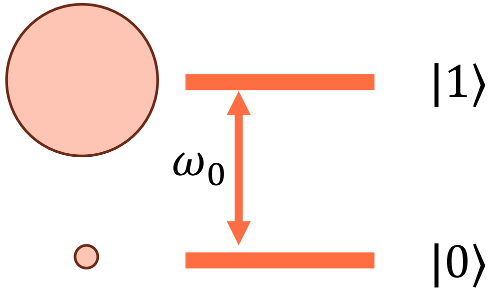
Rube Goldberg
machine (!)

Rydberg atoms: an analog quantum computer

We want to realize

$$H(t) = \frac{t}{t_f}\left(-\delta \sum_i \hat{n}_i + U \sum_{i,j\in E} \hat{n}_i \hat{n}_j\right) + \left(1 - \frac{t}{t_f}\right)\left(\Omega \sum_i \hat{\sigma}_i^x\right)$$

Rydberg atoms: artificial system that realizes a similar Hamiltonian



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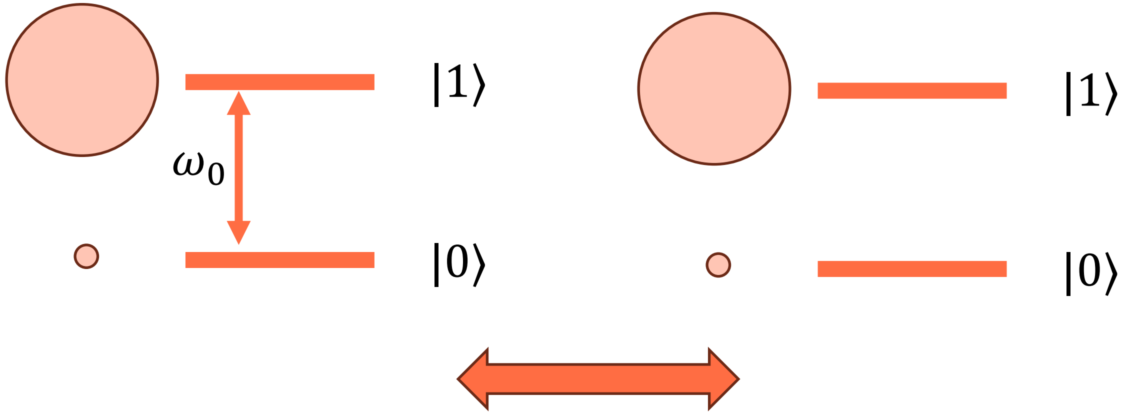

van der Waals interaction $\propto 1/r^6$

Review: Browaeys & Lahaye 2020

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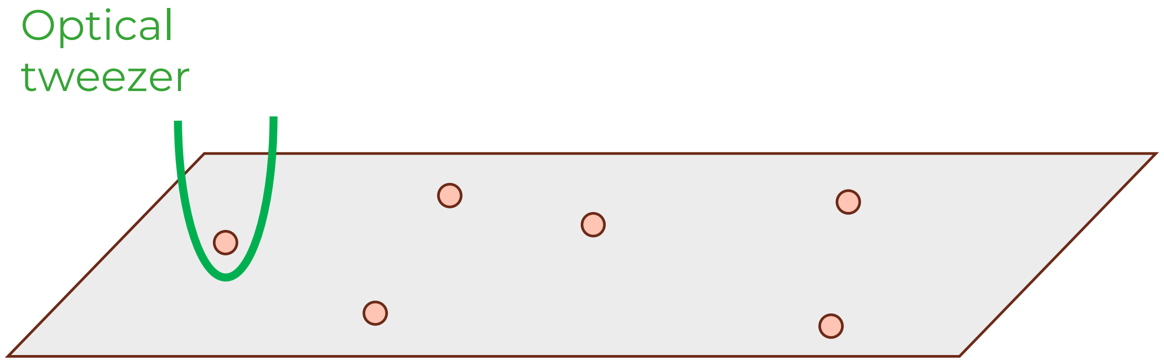

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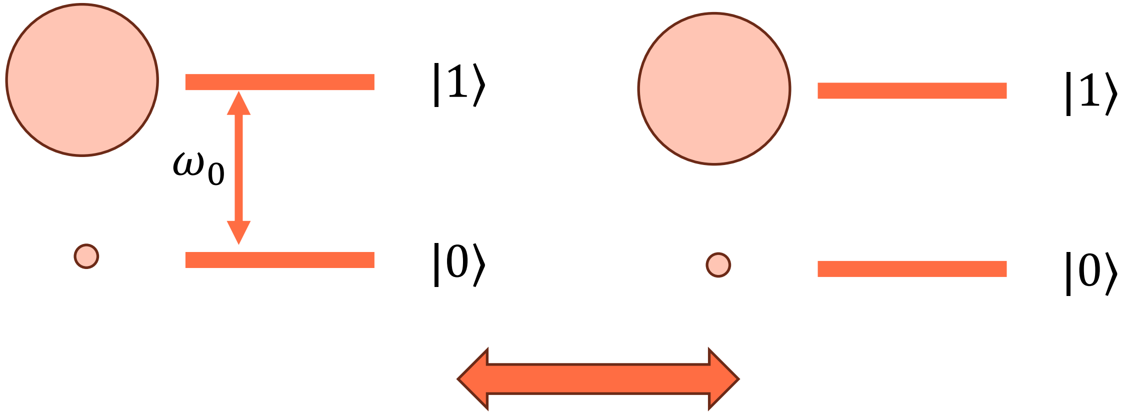

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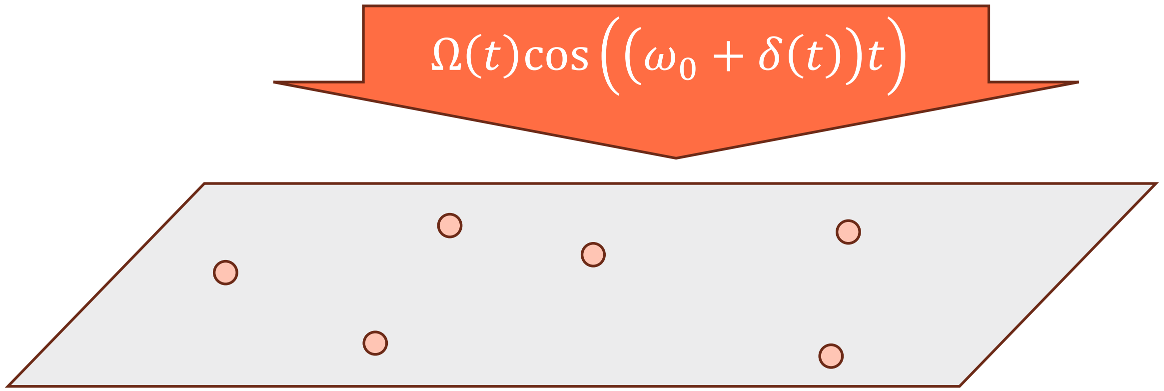

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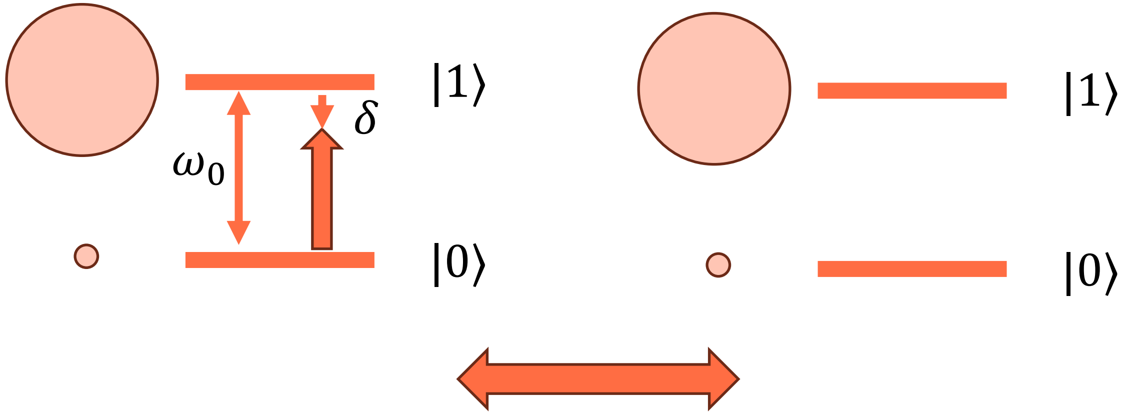

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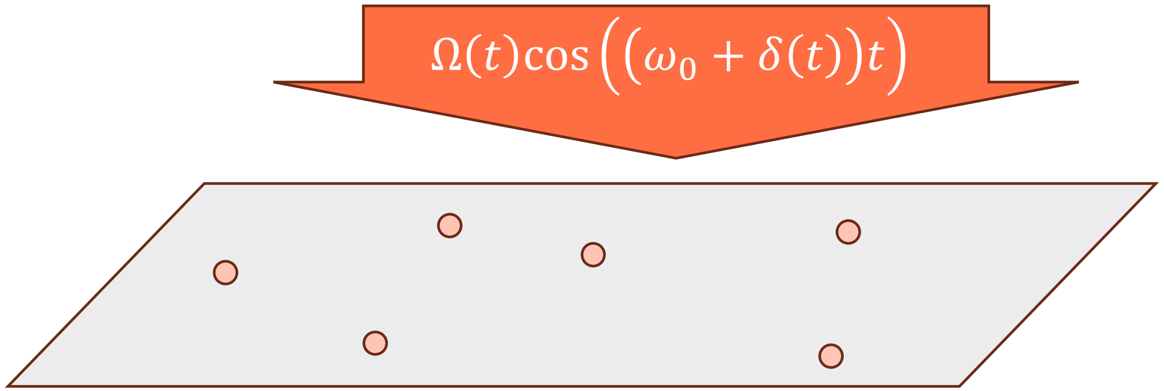

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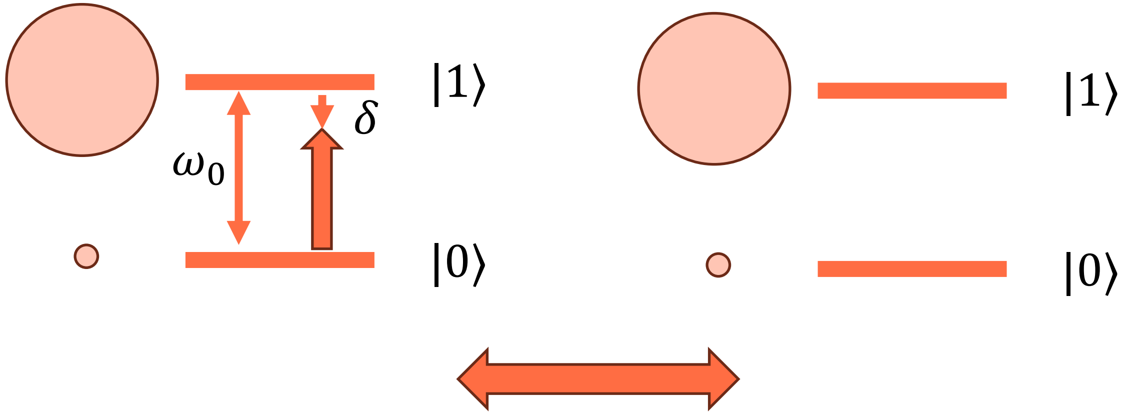
detuning **vdW** **Rabi**



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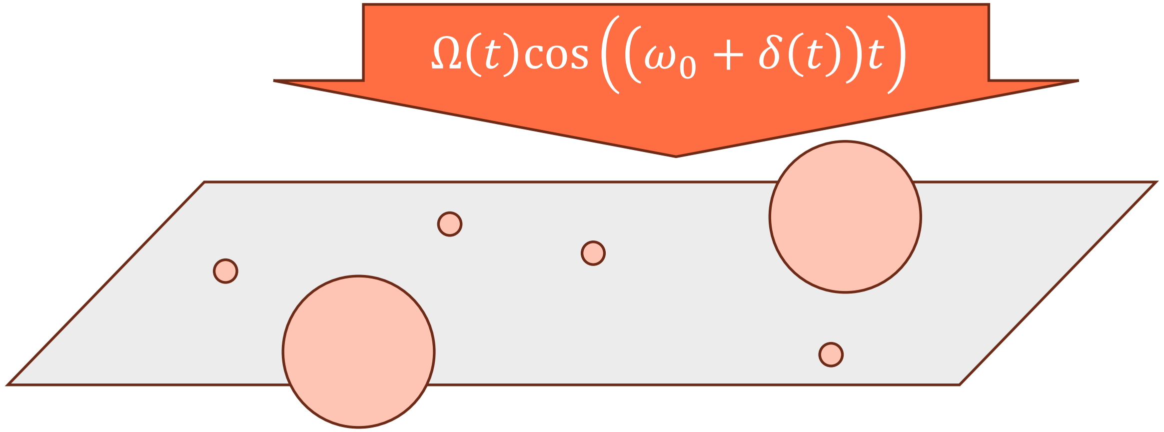

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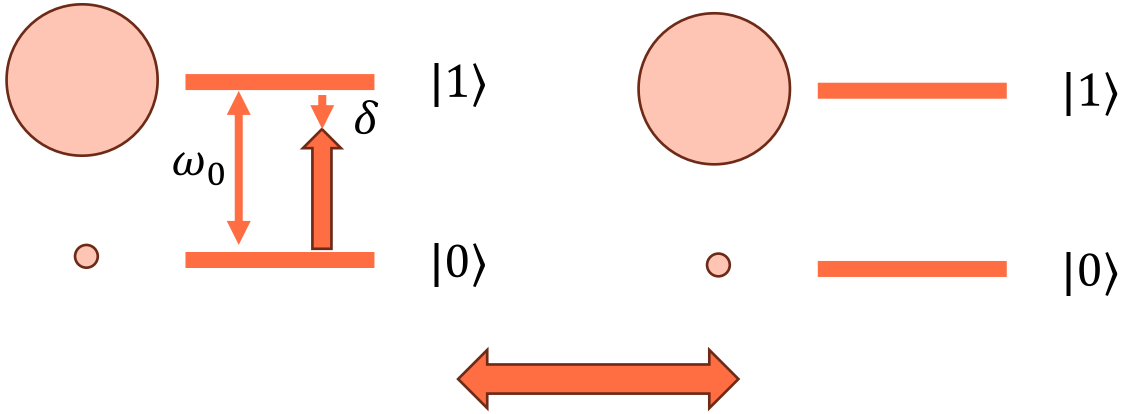

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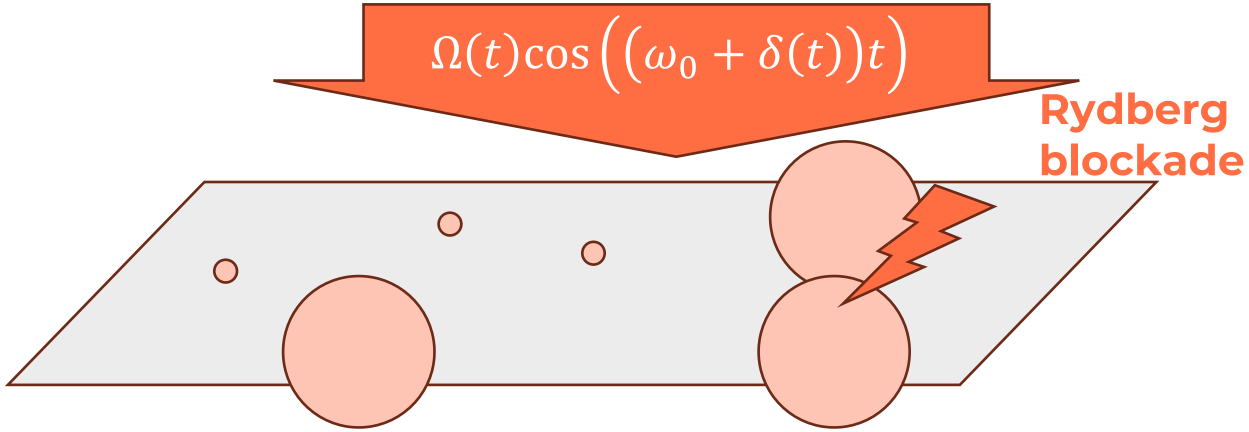

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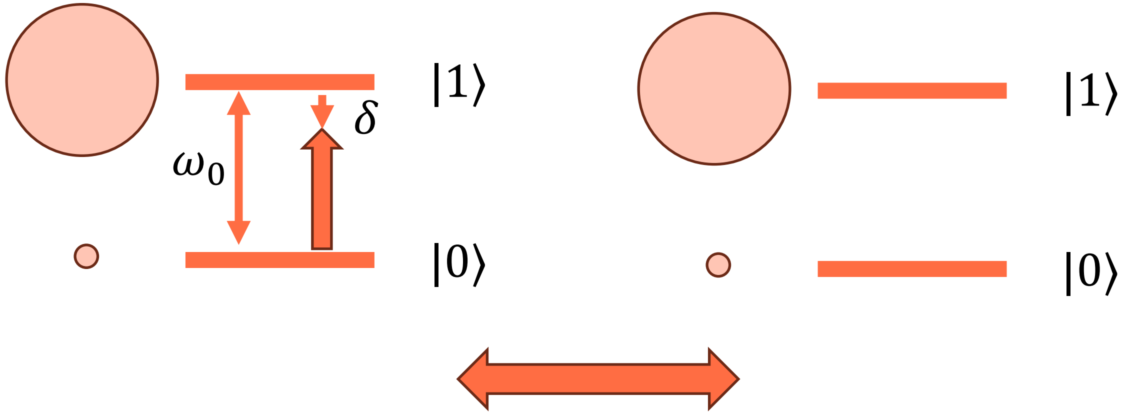

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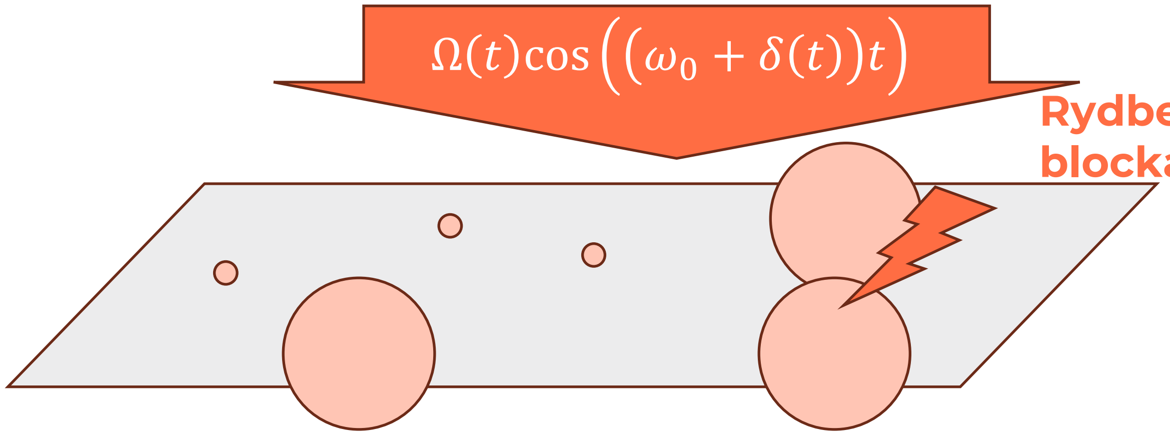

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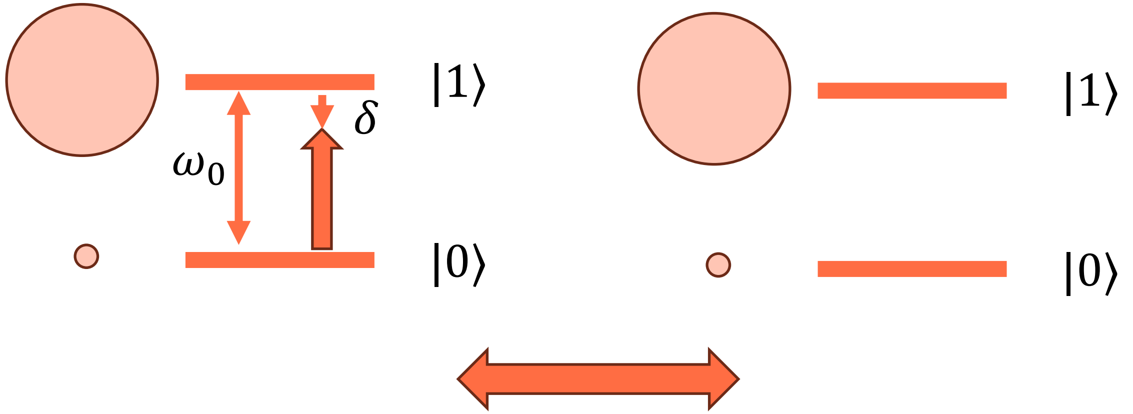

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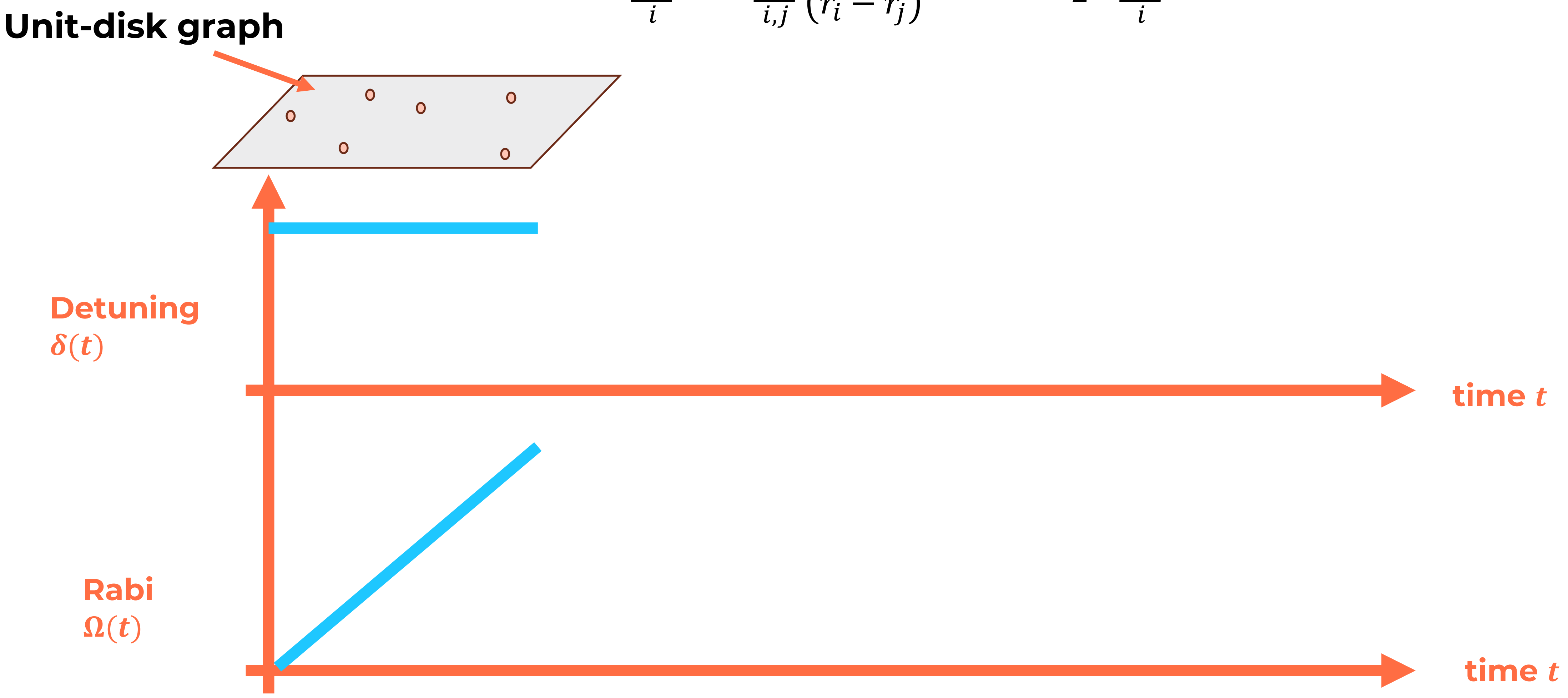
detuning **vdW** **Rabi**

Differences: interaction term

- cannot turn vdW on/off
- $\frac{1}{r^6}$ dependence

Summary: analog quantum computation with Rydberg atoms

$$H_{\text{Ryd}} = -\delta(t)\sum_i \hat{n}_i + \sum_{i,j} \frac{C}{\left(r_i - r_j\right)^6}\hat{n}_i\hat{n}_j + \frac{\Omega(t)}{2}\sum_i \hat{\sigma}_i^x$$



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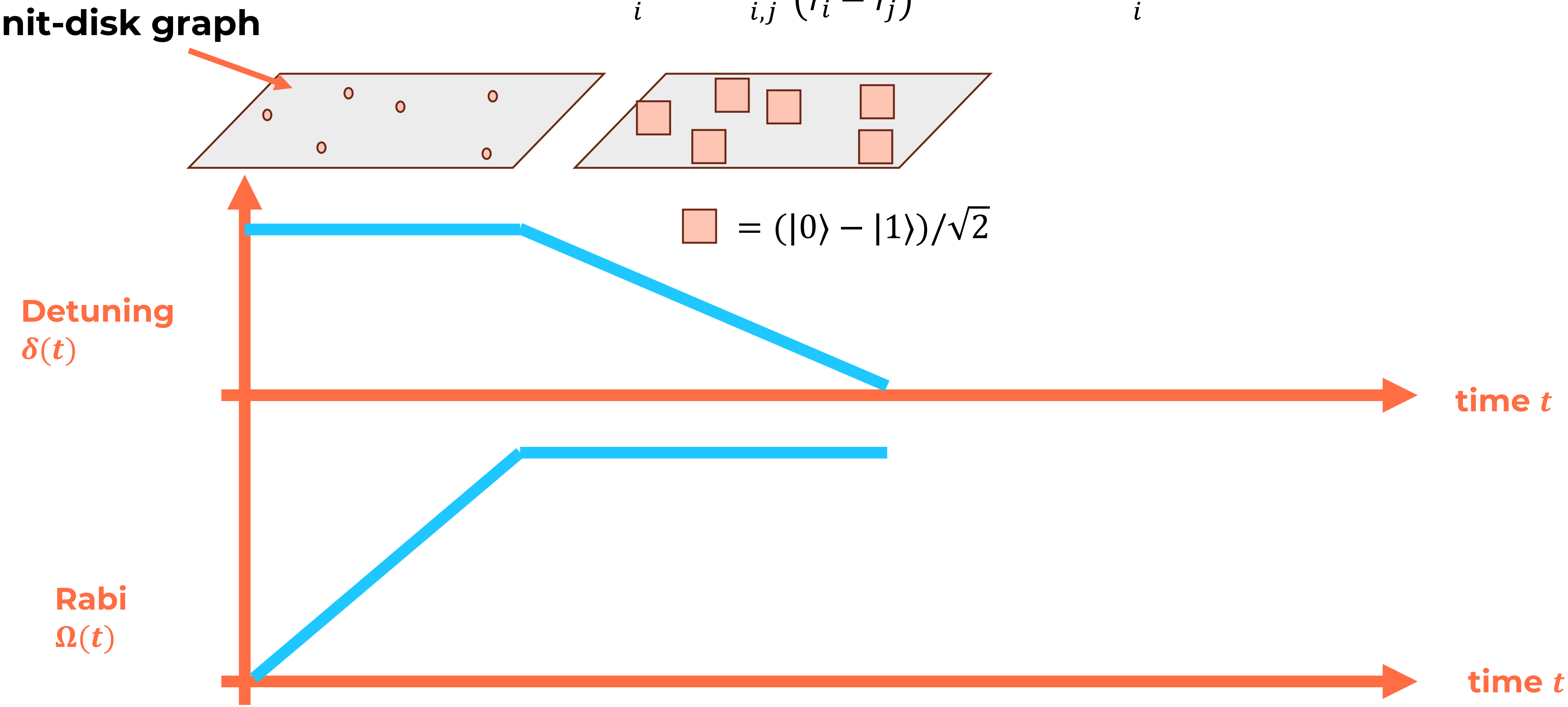

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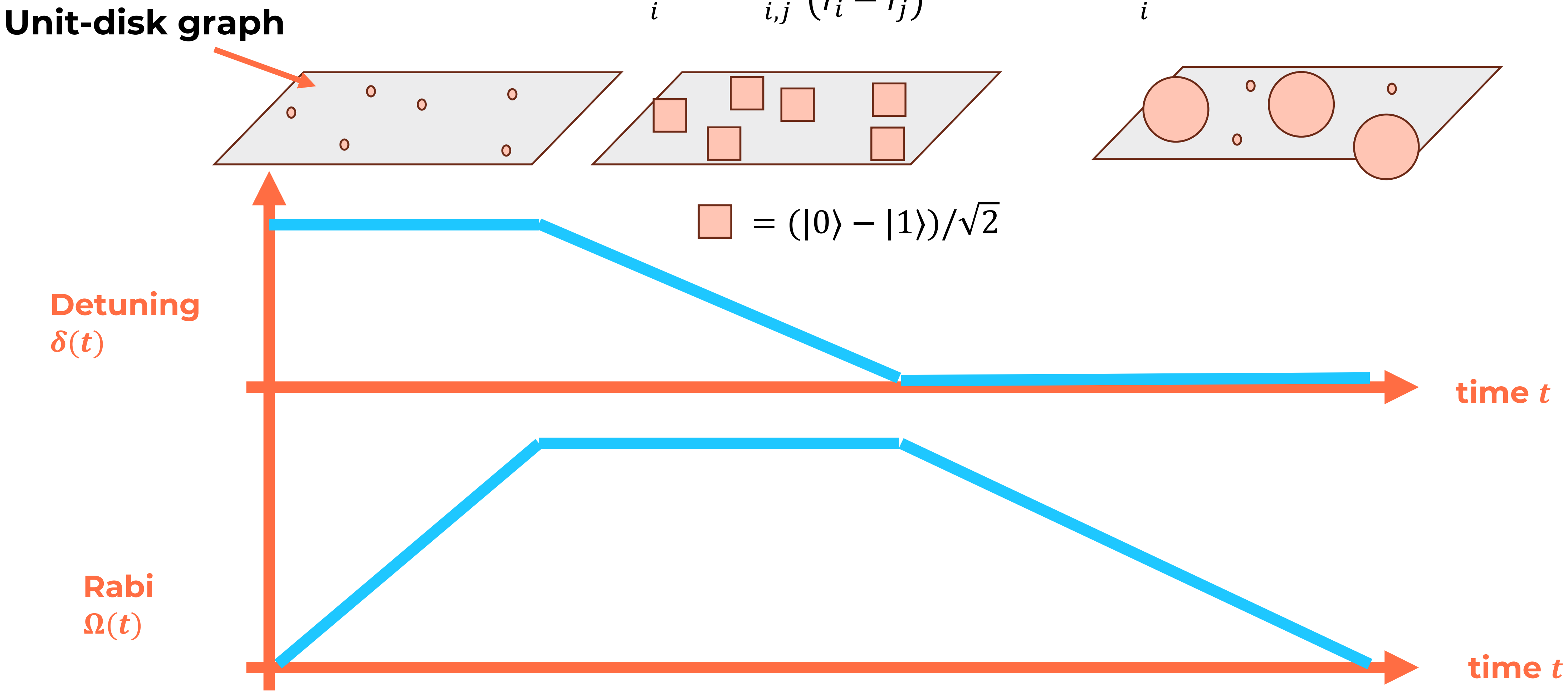

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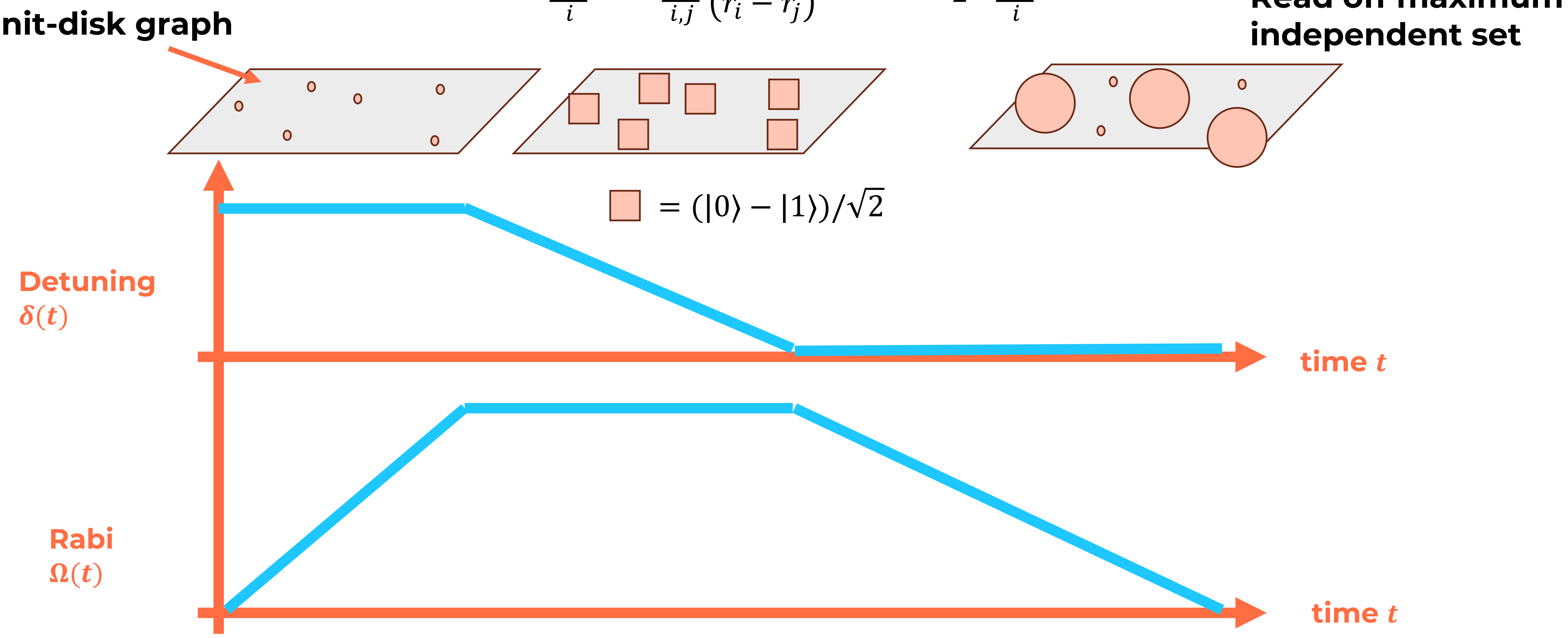
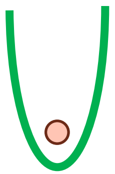


2 The challenge of decoherence

Many things can go wrong!

$$H_{\text{Ryd}} = -\delta(t)\sum_i \hat{n}_i + \sum_{i,j}\frac{C}{(r_i - r_j)^6}\hat{n}_i\hat{n}_j + \frac{\Omega(t)}{2}\sum_i \hat{\sigma}_i^x$$

Optical tweezer



Heisenberg uncertainty: $r + \Delta r$

$$\Delta E_{vdW} \propto \frac{C}{(r_i - r_j)^7}(\Delta r_i + \Delta r_j)$$

Systematic (calibration) error: $\tilde{\delta}(t)$
+
Decoherence
$\boldsymbol{\tilde{\delta}(t) + \Delta\delta(t)}$

Systematic (calibration) error: $\tilde{\Omega}(t)$
+
Decoherence
$\boldsymbol{\tilde{\Omega}(t) + \Delta\Omega(t)}$

Irreversible processes: accumulate with time

+ Readout errors

+ intrinsic limitations
cannot turn vdW on/off
$\frac{1}{r^6}$ dependence
need long annealing time!

Can we make a quantitative prediction of the approximation ratio?

Numerical simulation of the analog computation

Simplified (but realistic) error model with

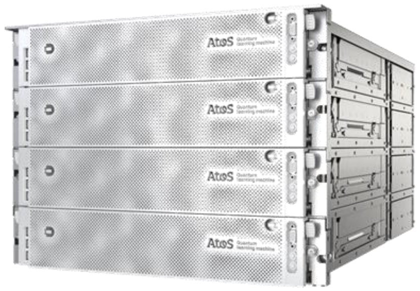
- **Readout noise**.

- **Dephasing white noise**: Lindblad equation

$$\frac{d\rho}{dt} = -i[H(t),\rho] - \frac{\gamma}{2}\sum_i \{n_i,\rho\} - 2n_i\rho n_i$$

  - Solved with trajectories method (Dalibard, Castin & Mølmer 1992)

Eviden Qaptiva compact
19" HPC appliance
NUMA architecture:
up to 32 Tb memory



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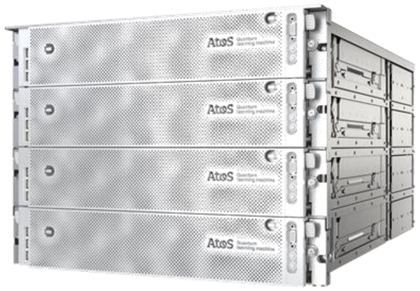

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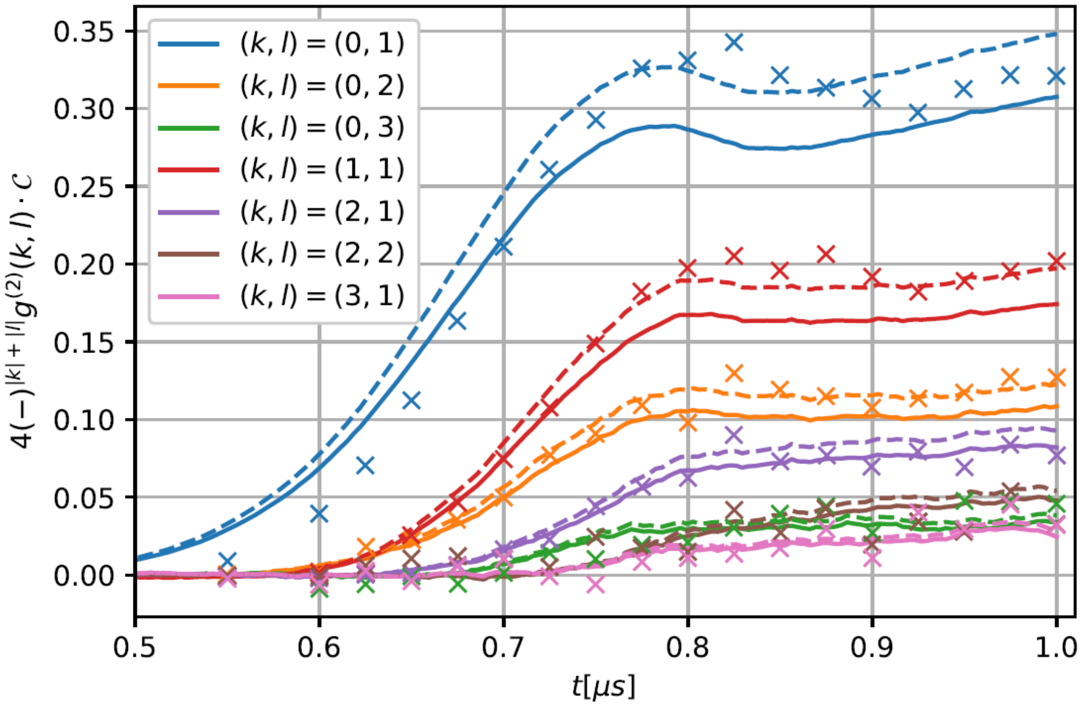
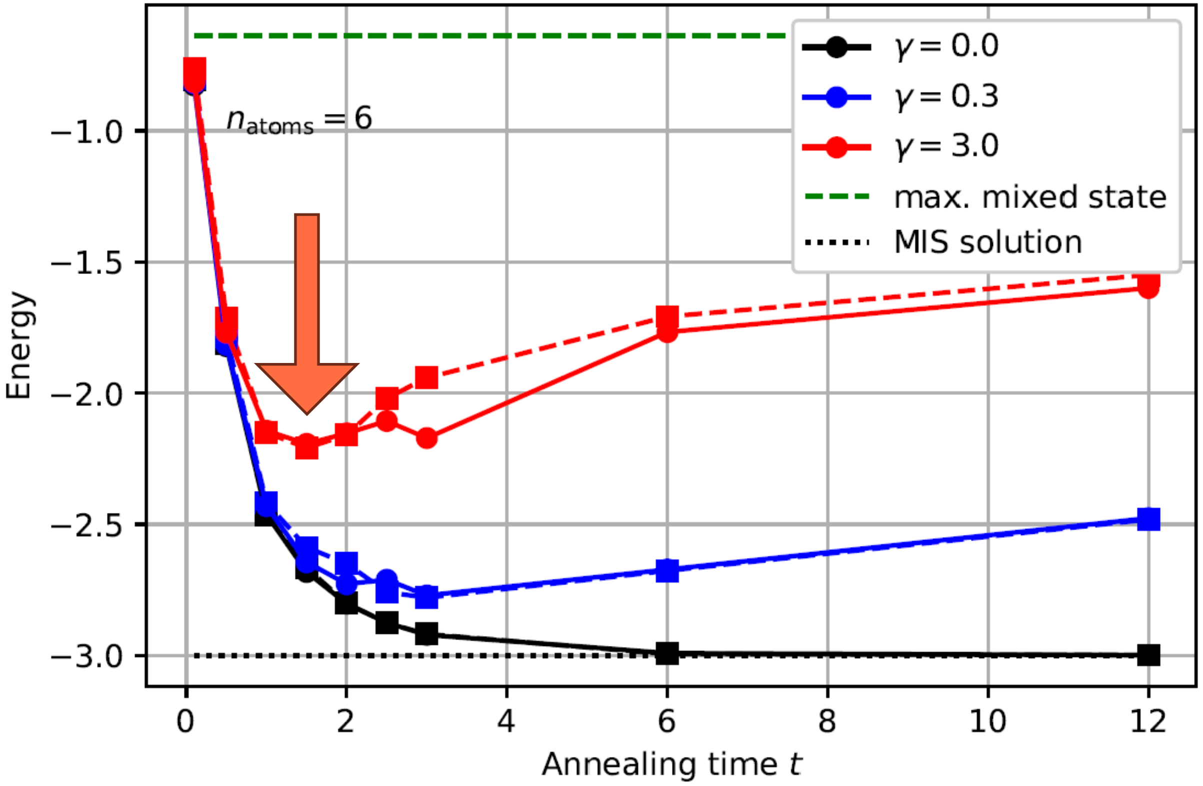
Validation for $\gamma = 3.0$ (exp: Lienhard '18)



Note: today's experiments: $\gamma = 0.3$ or even less!

Not too short, nor too long!



- **Noiseless case ($\gamma = 0$):** the longer, the better (adiabatic theorem)

- **Noisy case:** longer evolution = longer exposure to noise

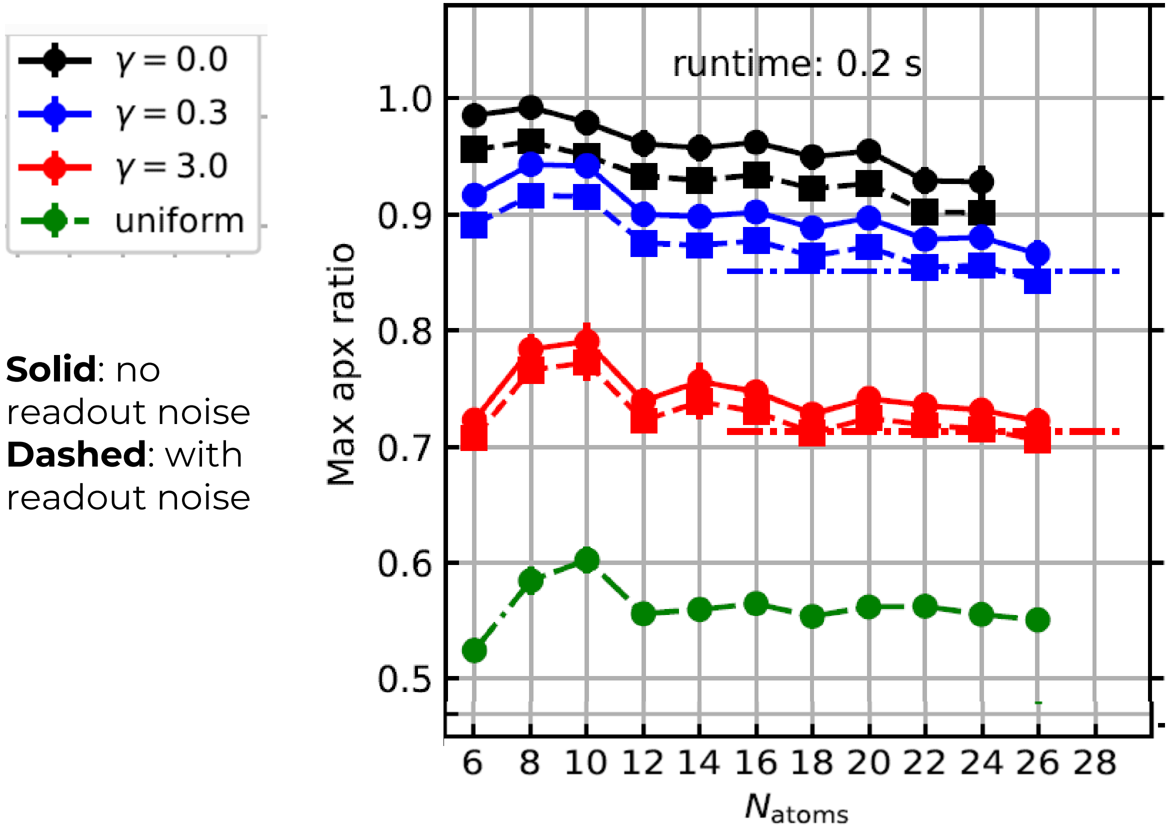
In the following:

- work at optimal time
- work in “IS” subspace (dashed lines)

Noise & size dependence

Serret, Marchand, TA, PRA 2020

Dependence of expected apx ratio α_{avg} on noise γ and graph size (N_{atoms})?



Solid: no readout noise
Dashed: with readout noise

- **Noise γ :**
  - Noise degrades α_{avg}
  - Even at $\gamma = 0$, $\alpha_{\text{avg}} < 1$ (imperfect annealing schedule: VdW interactions…)

- **Size N_{atoms} :**
  - Decreasing, then stable
  - Stable earlier for higher γ

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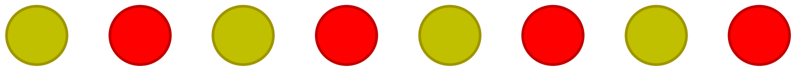
Correlations & decoherence

Serret, Marchand, TA, PRA 2020

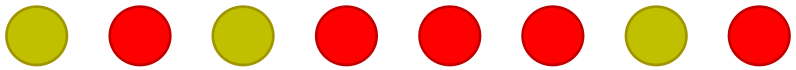
- Simple example: 1D chain



MIS

$z_0 = 1$ $z_3 = -1$

- Correlation function: $\langle z_i z_j \rangle$
  - Perfect “MIS” (antiferromagnetic) state: $\langle z_{2i} z_{2j} \rangle = 1, \langle z_{2i} z_{2j+1} \rangle = -1$
  - In the presence of decoherence: defects



Generically: $E(|\langle z_i z_{i+r} \rangle|) \propto e^{-r/\xi}$
with ξ correlation length.

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Serret, Marchand, TA, PRA 2020

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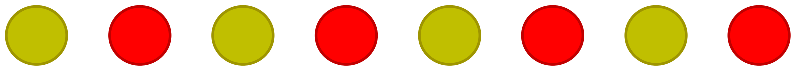

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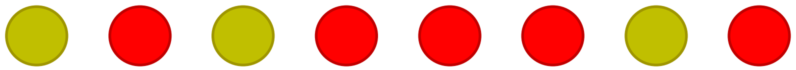

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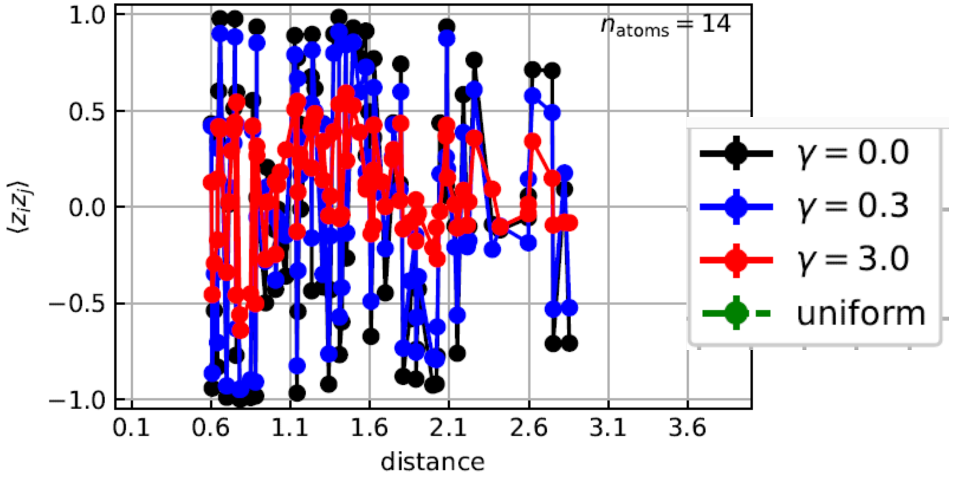
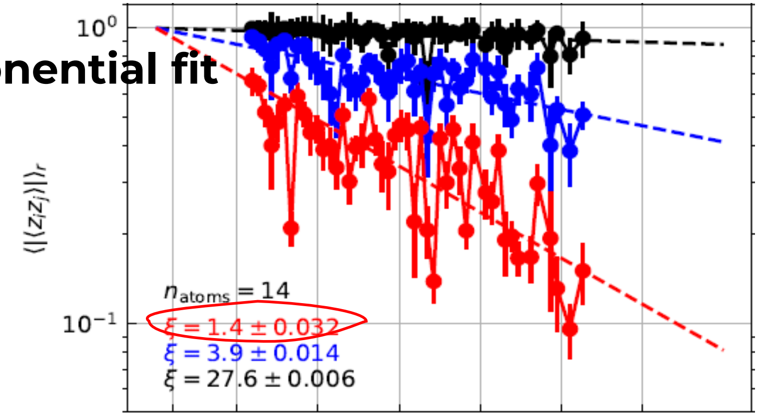
On our (2D) graphs:



Exponential fit
$e^{-r/\xi}$



distance

- ξ decreases with increasing noise
- Can check: Correlation length is roughly independent of system size

Correlation length & approximation ratio

Can we relate the quality of a quantum algorithm with the correlation length?

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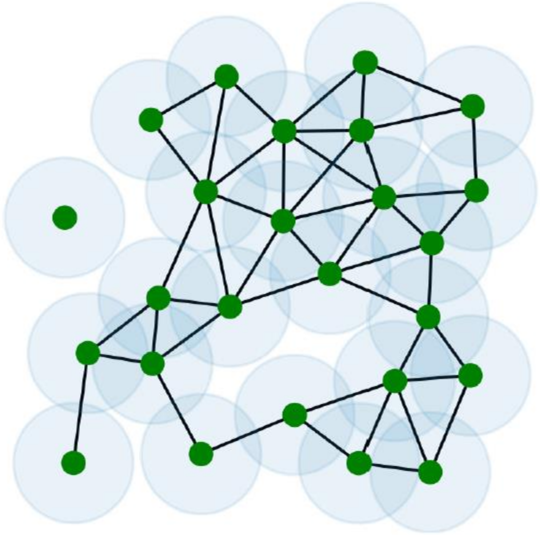
Classical approaches... also have a kind of 'correlation length'!

All classical approaches:

"Divide and conquer"

Split the problem in smaller pieces and solve the pieces exactly



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For instance:

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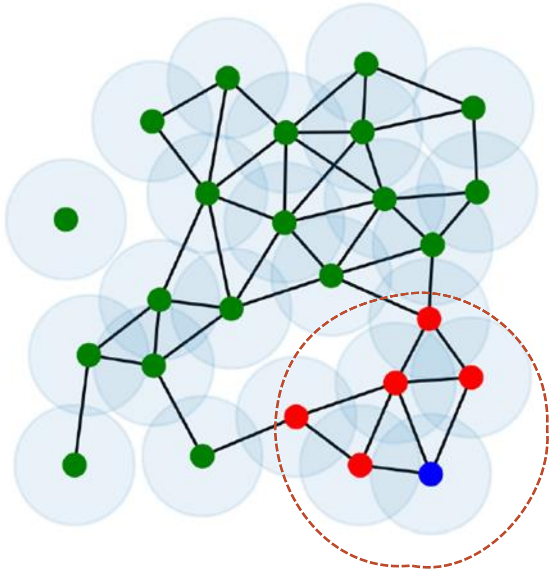
- Solve exactly within 'distance' d



distance d

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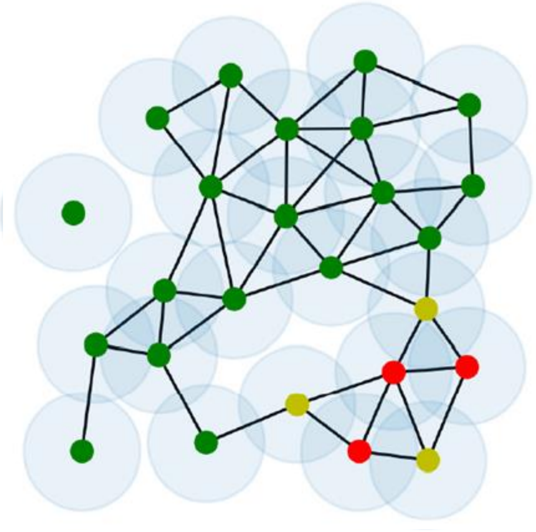
- Remove subgraph (+connected vertices) from available vertices



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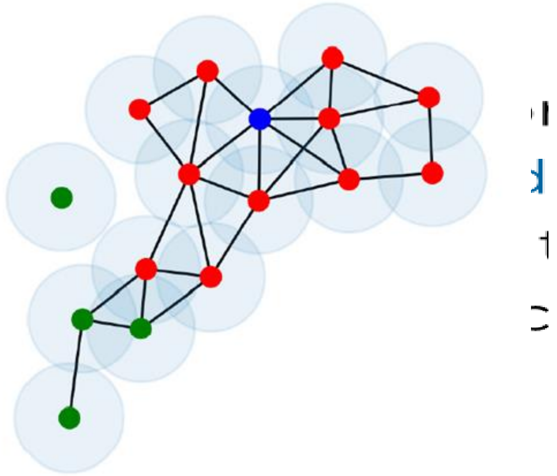
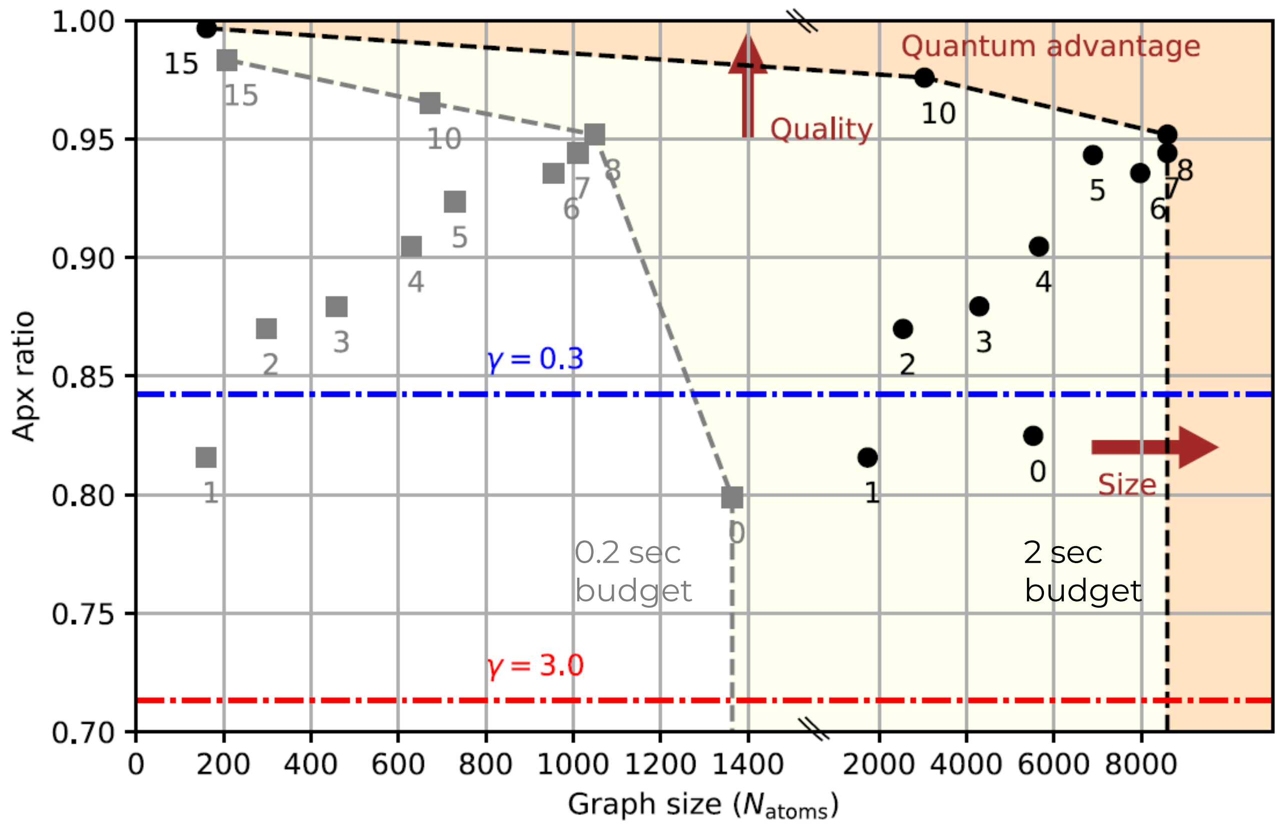
- Solve exactly within 'distance' d
- Append sub-solution to solution
- Remove subgraph (+connected vertices) from available vertices
- Iterate



A roadmap for quantum 'advantage'



Two ways to overperform the classical heuristic:

- **Go faster for bigger systems** (beat the exponential): "Size"

~ 1,000 atoms for 0.2 secs

~ 8,000 atoms for 2 secs

Scaling of repetition rate?

- **Reach higher approximation ratios**: "Quality"

~ e.g, 0.97 for 2000 atoms (2 secs)

Better hardware?

(better readout, lower noise, …)

(Cf circular Rydberg atoms, Nguyen et al 2018)

Better algorithms?

QAOA (digital), QA (better resource Hamiltonian), …

Or solve more difficult problems!

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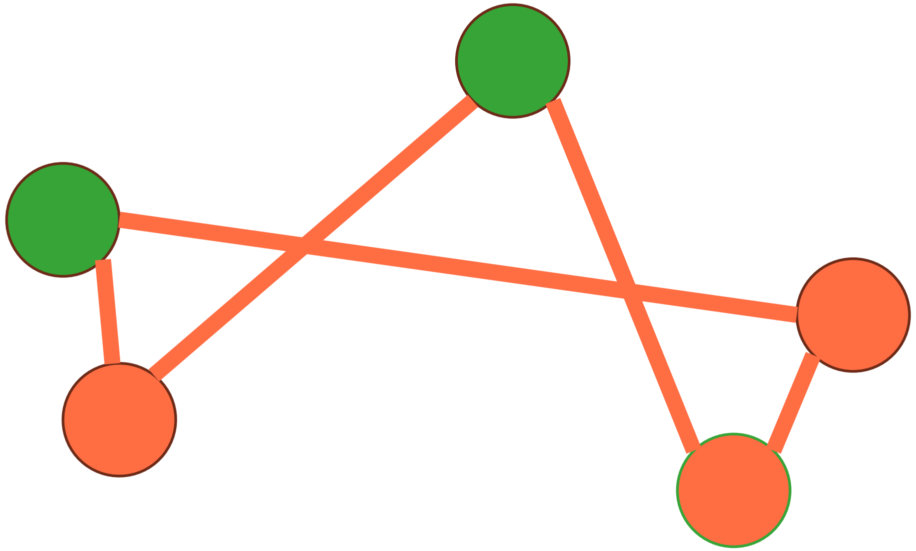
3 Towards more general graphs… and applications

What about non-unit-disk graphs?

If G not unit disk graph: Harder problem… better candidate for quantum acceleration!?



Nguyen et al 2022

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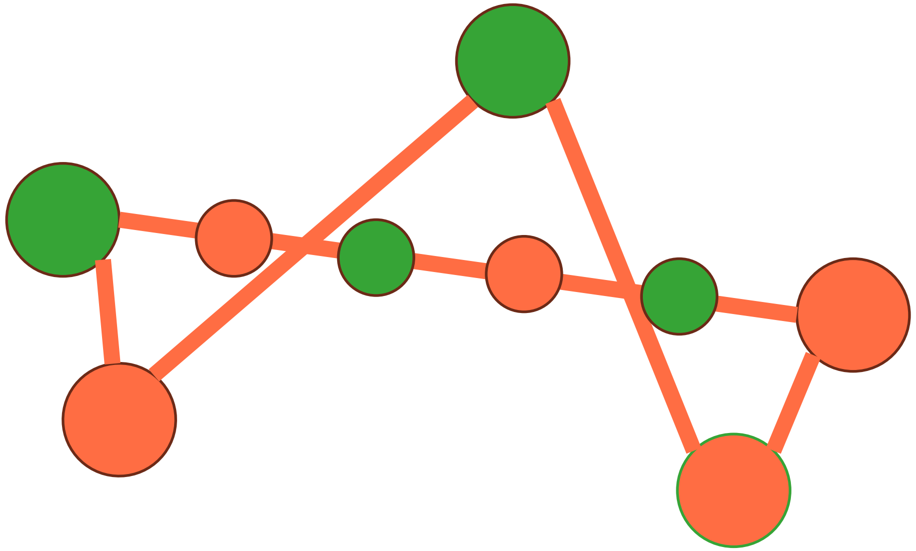

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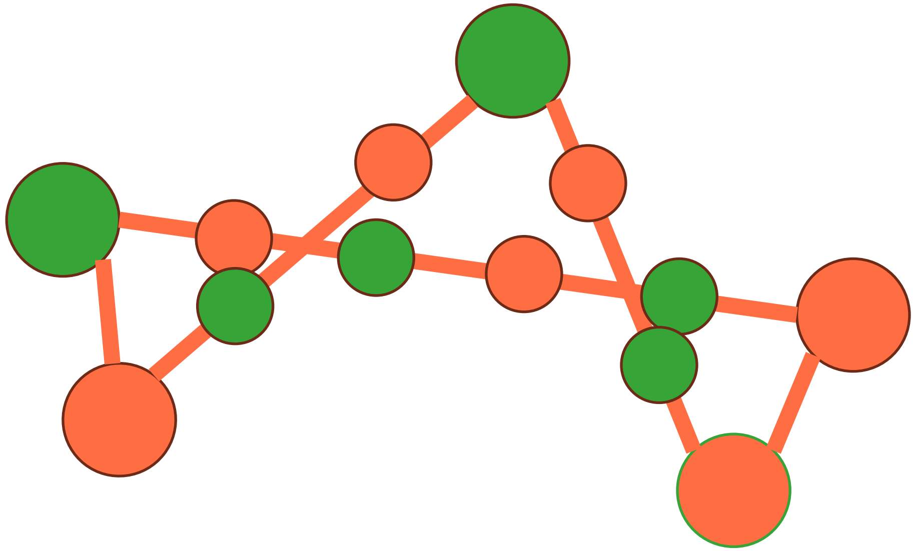

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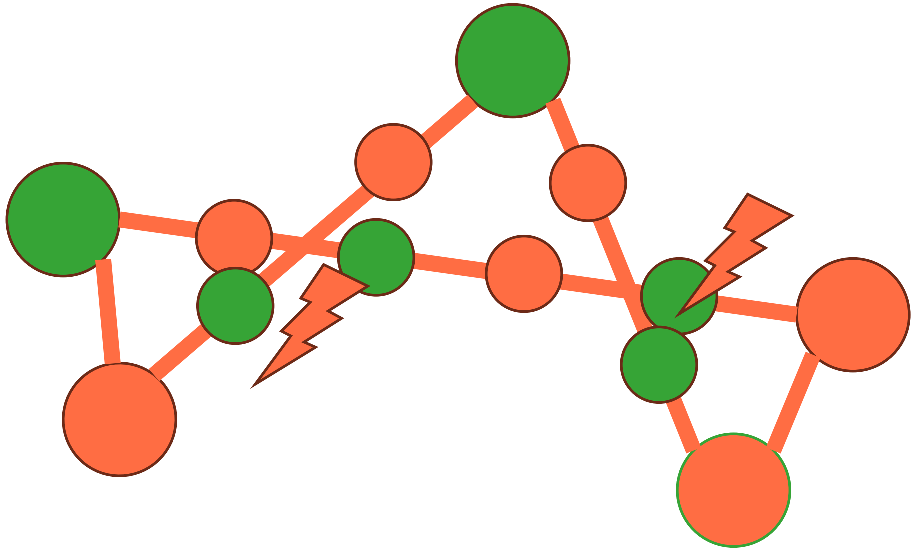

Crossings of edges need to be handled properly!

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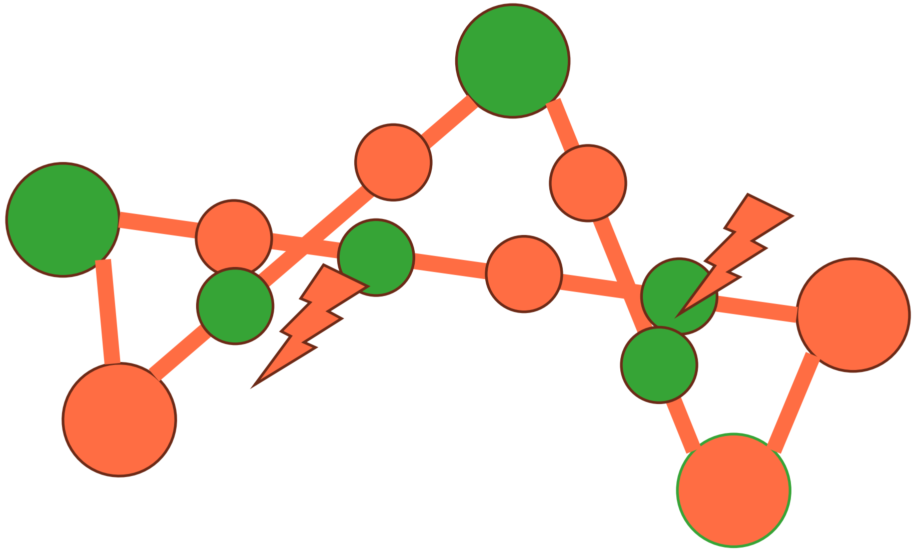

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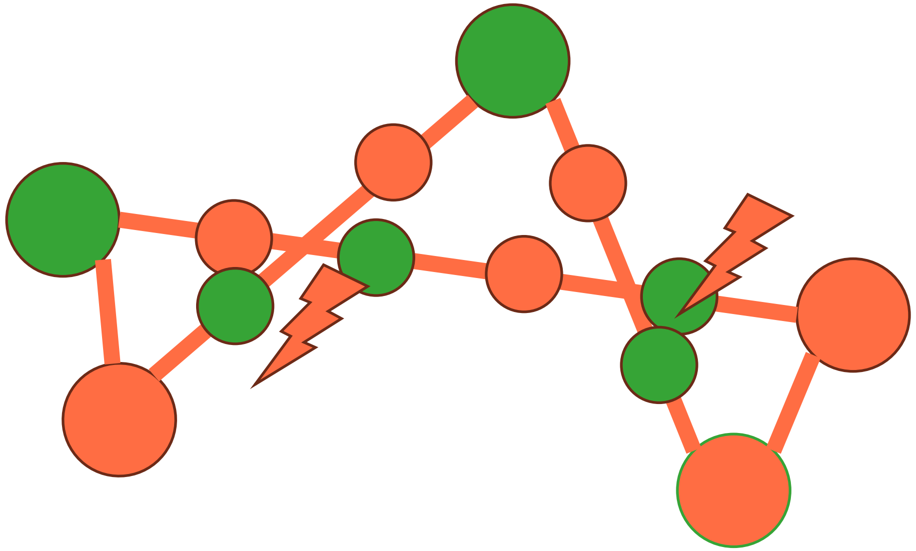

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Comes at a price:

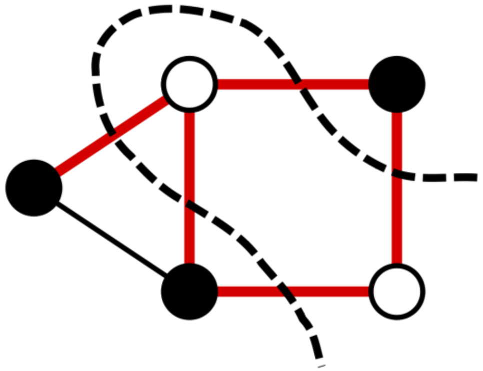
- Number of vertices (atoms): **quadratic increase: $O(N^2)$ atoms!**
- Needs local detuning $\delta_i(t)$!

What about other combinatorial optimization problems?

At the cost of adding vertices, can solve any « **quadratic, unconstrained optimization problem** » (QUBO):

$$H = \sum_{ij} J_{ij} s_i s_j + \sum_i h_i s_i \qquad s_i \in \{-1,1\}$$

For instance, **MaxCut problem**:



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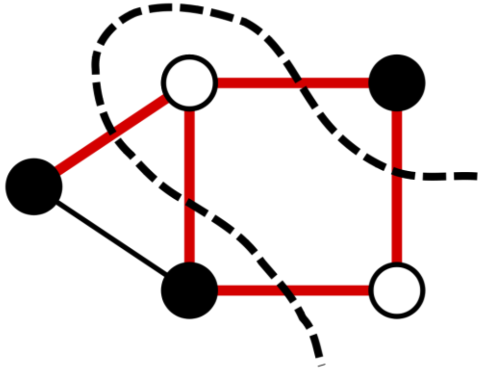

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Other example: **number factoring**.

Hard problems (spin glass physics)

What about... solving fermionic problems?

Hubbard model: prototypical correlated electron problem.

- Quantum simulation by cold fermionic atoms!
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Yes (Jordan-Wigner, etc...), but Hamiltonian is very different from Rydberg Hamitonian!

$$c_i^\dagger c_j \to X_i Z_{i+1} \cdots Z_{j-1} X_j \ \ (+\cdots)$$

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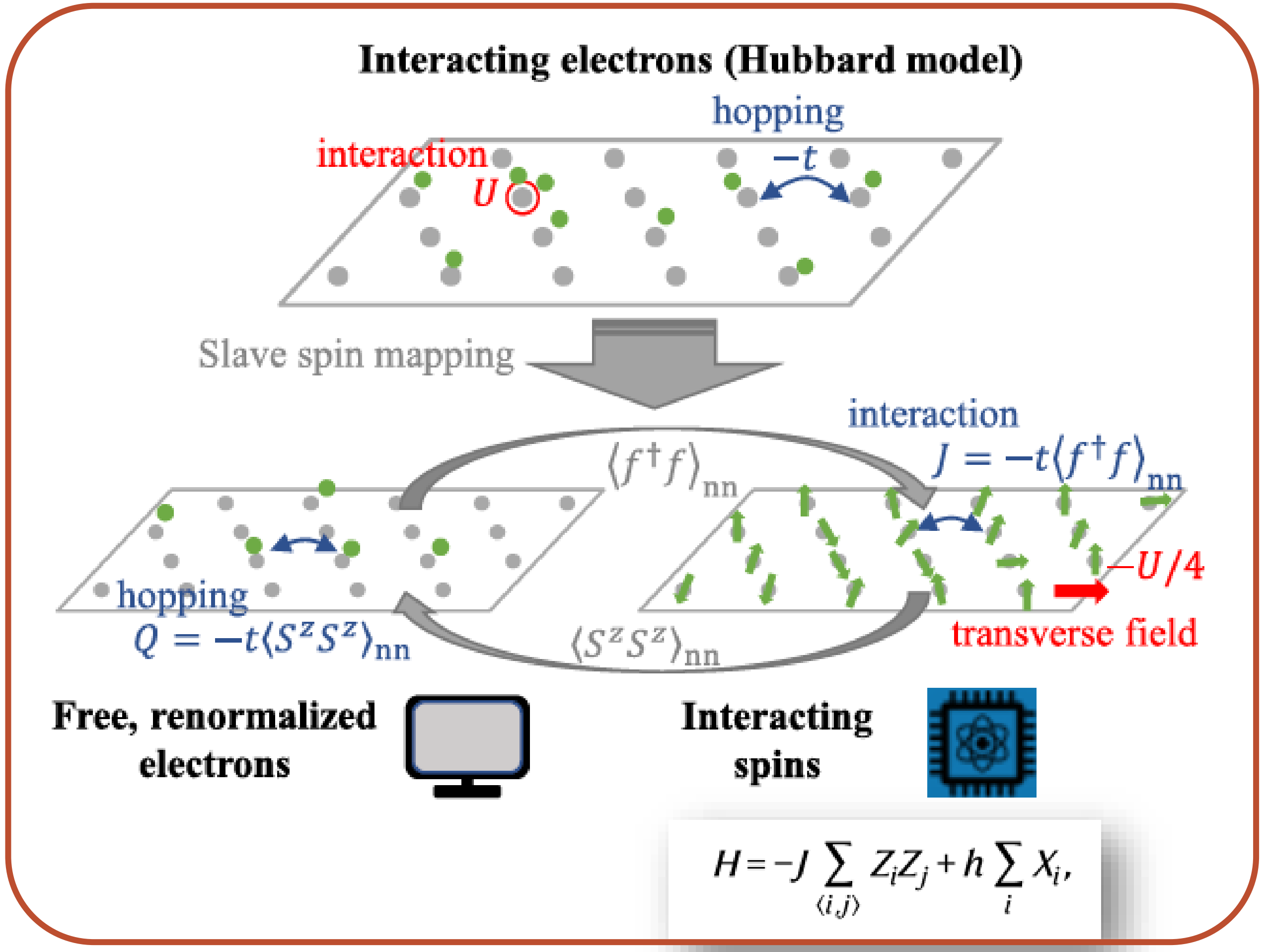
Idea: use a slave-spin mapping: $c_{i\sigma}^\dagger = f_{i\sigma}^\dagger Z_i$

+ Mean-field decoupling

de' Medici 2005
Rüegg et al 2010
Hassan 2010



Very close to Rydberg Hamiltonian!

Using Rydberg atoms to deal with the spin model

Michel, Henriet, Domain, Browaeys, TA, 2312.08065

Effective model: Transverse Field Ising model (TFIM):

$$H_{\mathrm{s}}^{\mathcal{C}} = \sum_{i,j\in\mathcal{C}} J_{ij} S_i^z S_j^z + \frac{U}{4}\sum_{i\in\mathcal{C}} S_i^x + \sum_{i\in\mathcal{C}} h_i S_i^z,$$

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$$\hat{H}_{\mathrm{Rydberg}} = \sum_{i\neq j} \frac{C_6}{|\mathbf{r}_i - \mathbf{r}_j|^6}\hat{n}_i\hat{n}_j + \frac{\hbar\Omega(\tau)}{2}\sum_i \hat{S}_i^x - \hbar\delta(\tau)\sum_i \hat{n}_i,$$

Main challenges:

- Optimize atoms positions to reproduce J_{ij}
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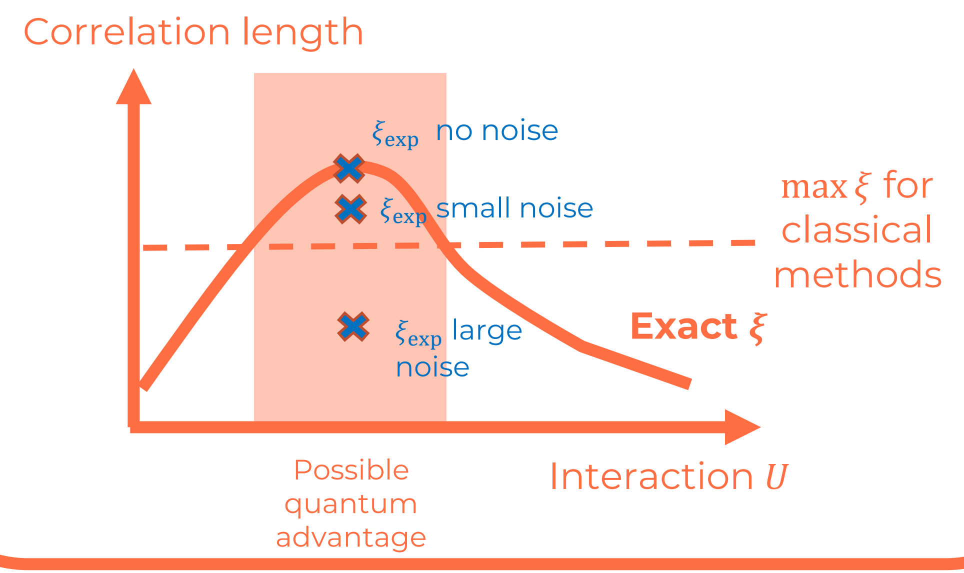
Hope for advantage w.r.t classical methods:



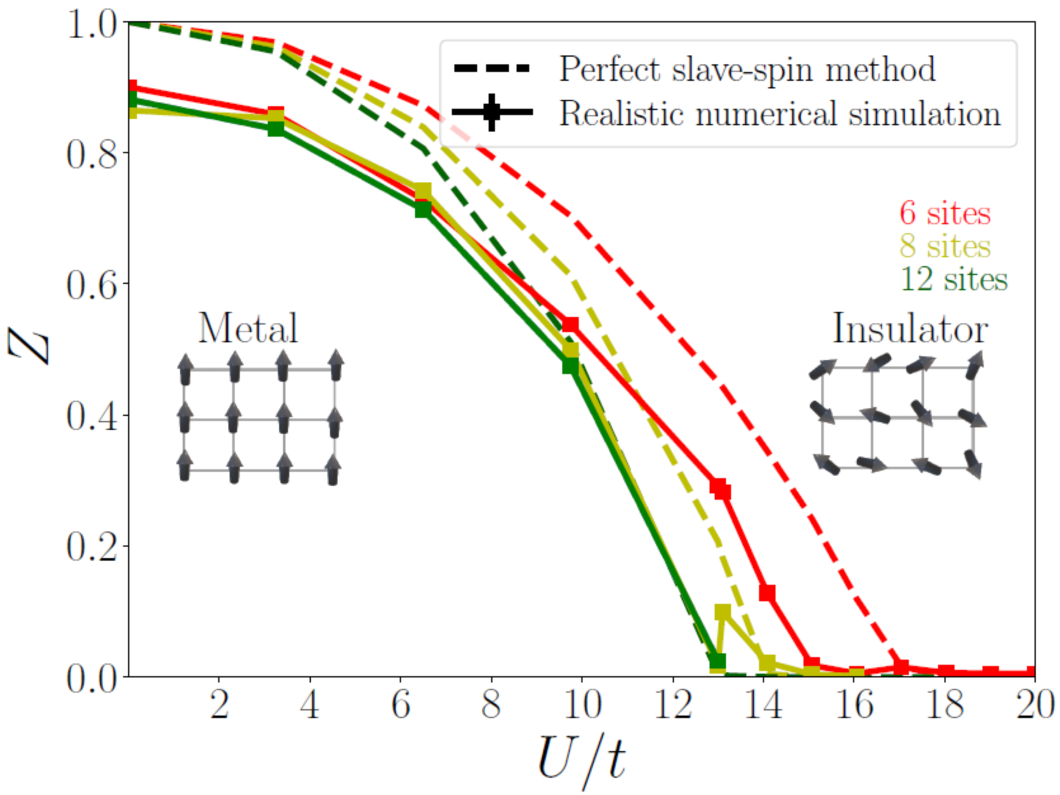
Mott physics with Rydberg atoms: (emulated) results

Michel et al, 2312.08065

Can we locate Mott transition?

… in the presence of noise.

Numerical emulation with realistic noise



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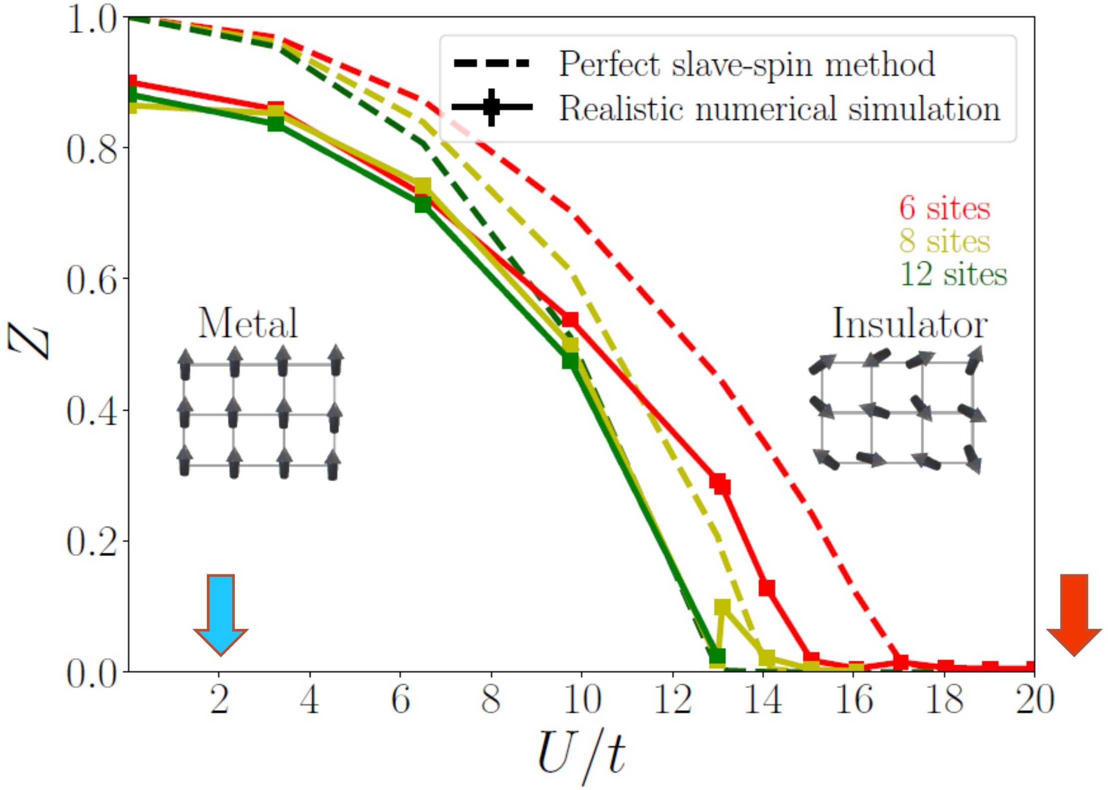
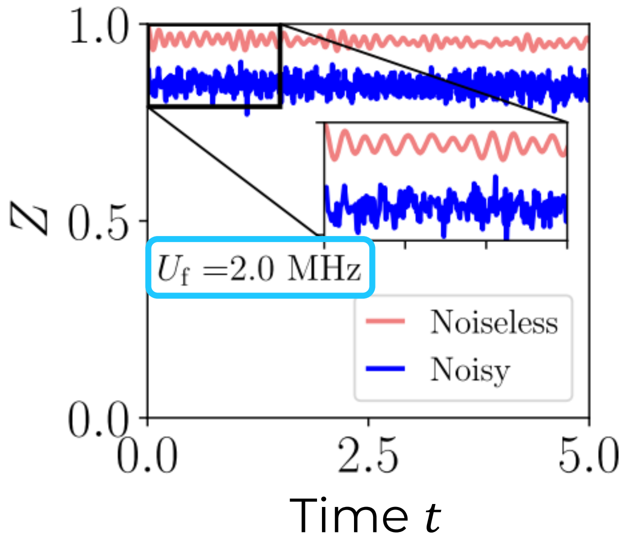
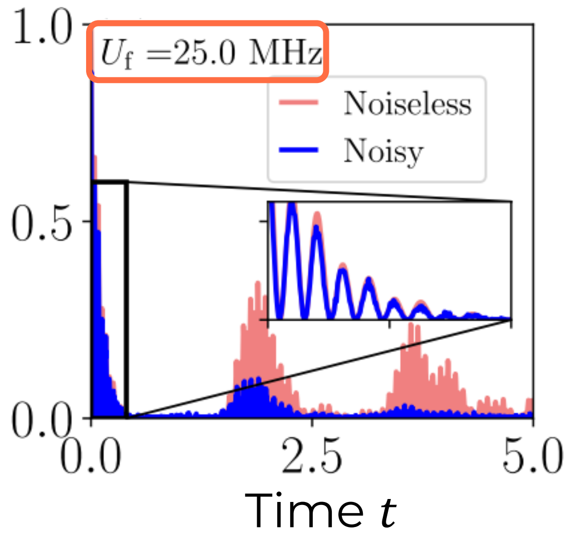


What about out-of-equilibrium?

Interaction **quench** of the Hubbard model: becomes quench of transverse field in TFIM





Ongoing experimental implementation!

Conclusions

Rydberg platforms

- straightforward mapping to specific combinatorial optimization problems

Decoherence limits correlation length

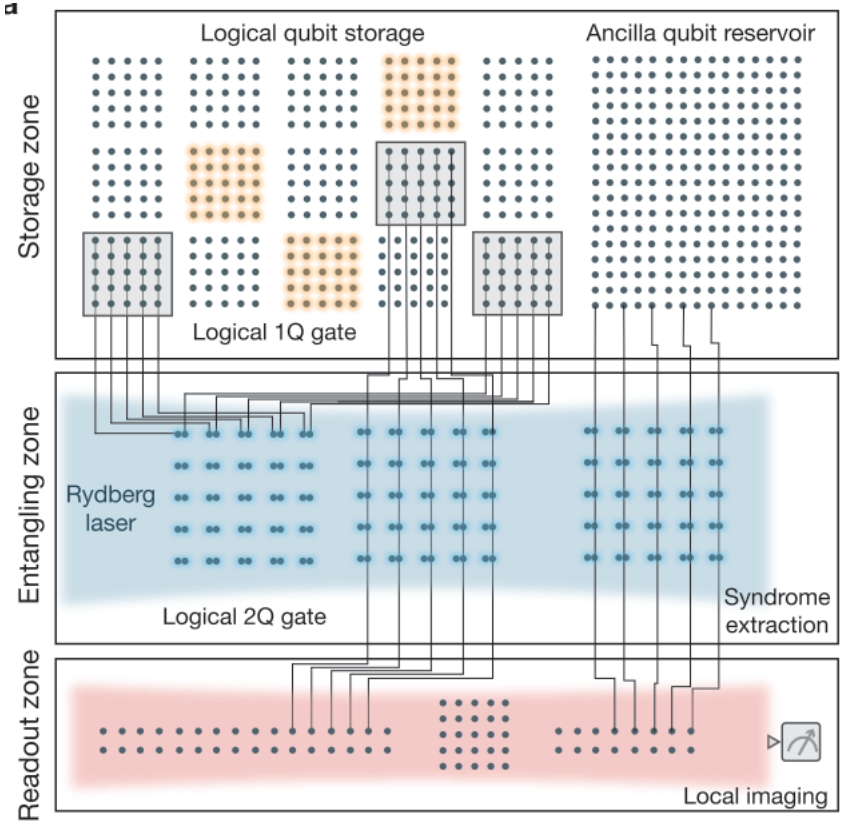
- Lesser success probability

Recent extensions

- More general graphs
- Fermionic problems
- And others
  - machine learning: quantum evolution kernel Henry et al 2021

Recent breakthrough:

quantum error correction architecture (Bluvstein et al 2024)



Thank you!

thomas.ayral@eviden.com