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Cooling and Tunnelling of Atoms in a 2D Laser Field.

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Abstract. – We present a quantum treatment of Sisyphus cooling in two dimensions. The steady state is given by the populations of the energy eigenstates for the atomic motion in the potential created by the atom-light interaction. In addition to characterizing the momentum and position distributions, we show that resonant tunnelling between adjacent optical potential wells leads to a sizable variation of the lowest-energy state population, as a function of the field parameters.

Laser cooling of atoms has been thoroughly studied experimentally as well as theoretically during the last decade [1]. In the so-called optical molasses, the minimum temperatures obtained correspond to r.m.s. momenta of the order of a few photon momenta $\hbar k$ [2]. Consequently, the atomic de Broglie wavelength becomes of the order of the optical wavelength λ, and one has to treat the atomic motion in the light field quantum mechanically. In a one-dimensional geometry (1D), the discrete energy levels related to the quantized atomic motion inside the potential wells have been calculated [3] and observed experimentally [4, 5]. Recently, experimental evidence for these quantum levels in a 2D [6] and a 3D [7] geometry has also been obtained.

We present here a quantum treatment of Sisyphus cooling [8] in a 2D laser configuration, for a transition between a ground state g and an excited state e with angular momenta $J_g = 1/2$ and $J_e = 3/2$. First we calculate the eigenstates of the Hamiltonian describing the atomic motion in the 2D periodic potential created by the light. We then derive the transfer rates between these eigenstates and their steady-state populations. Resonances are predicted in the populations of the lowest-energy levels as functions of laser intensity or detuning. This remarkable phenomenon is due to tunnelling of the atom between adjacent optical potential wells, whose minima are separated by a distance $\lambda/2$. This effect was not

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present in the 1D situations studied so far. Its observation would constitute a novel demonstration of the quantum nature of atomic motion in optical molasses.

The field configuration consists of two standing waves along the x and y axes, linearly polarized in the y and x directions. We restrict ourselves to laser fields with a saturation parameter $s_0 \ll 1$, which are known to lead to the lowest temperatures. The parameter $s_0 = 2\Omega^2/(\Gamma^2 + 4\delta^2)$ involves the Rabi frequency $\Omega = -dE_0/\hbar$ characterizing the coupling between the atomic dipole d and the field amplitude E_0 of each travelling wave, the natural width Γ of the atomic excited state and the detuning δ between the laser and atomic frequencies.

When expanded in the standard basis associated with the z-axis, the electric field has only σ_+ and σ_- components, since it lies in the (x, y)-plane. Consequently, after adiabatic elimination of the excited state and restriction to sufficiently low atomic velocities that the Doppler effect may be neglected, the reactive part of the atom-laser coupling is diagonal in the basis $|g_\pm\rangle = |g, m_z = \pm 1/2\rangle$ with elements corresponding to periodic potentials $U_\pm(x, y)$ (light shifts). Denoting the intensities of the $\sigma_\pm$ components of the field by $I_\pm(x, y)$ and using the squares of the appropriate Clebsch-Gordan coefficients, we find that the $U_\pm$'s are proportional to $I_\pm + (1/3)I_\mp$ and are equal to

$$U_\pm(x, y) = -\frac{U_0}{3}(\cos^2 kx + \cos^2 ky \pm \cos kx \cos ky \sin\alpha), \tag{1}$$

where α is the phase between the two standing waves and $U_0 = -4\hbar\delta s_0$.

Sisyphus cooling originates from the difference between U_+ and U_-, and from the correlation between light shifts and optical-pumping processes. For example, an atom in the state g_+ will, with maximum probability, be transferred to g_- by optical pumping at a position where I_- is large and I_+ small, corresponding to a top for U_+ and a valley for U_- if one has chosen $\delta < 0$ ($U_0 > 0$). This preferential transition from potential hills to valleys, corresponding to transfer of atomic potential energy into fluorescence field energy, is the dissipative element of the Sisyphus mechanism. Note that it does not take place if $\alpha = 0$, since in this case $U_+ \equiv U_-$. Such a sensitivity of multidimensional laser cooling to the relative phase of the standing waves forming the molasses has been predicted within a semi-classical treatment [9].

From now on we take $\alpha = \pi/2$. In fig. 1a) the variation of $U_+(x, y)$ is plotted along $y = 0$. Two types of minima are observed: the global minima $U_+ = -U_0$ in, *e.g.*, $(x, y) = (0, 0)$, and the local minima $U_+ = -U_0/3$ in, *e.g.*, $(\lambda/2, 0)$. The highest potential hills $U_+ = 0$ are located at $(n + 1/2, n' + 1/2)\lambda/2$ (field nodes). The topography of U_- is deduced from that

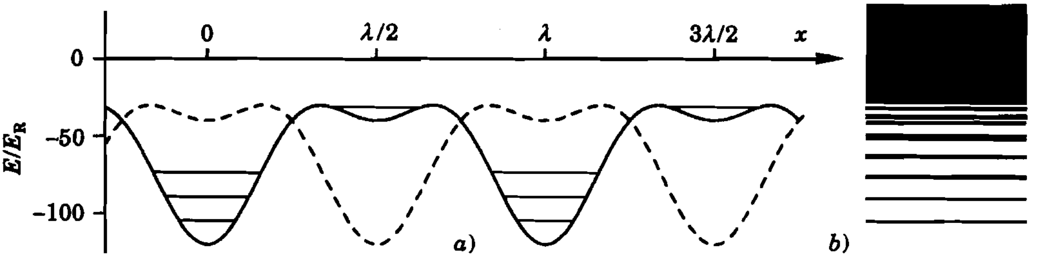

Fig. 1. – $U_0 = 120 E_R$. a) Variation of U_+ (solid line) and U_- (dotted line) along the line $y = 0$. The energy levels are obtained in the harmonic approximation at the bottom of the potential wells. b) Band energy spectrum calculated numerically.

of U_+ by a translation of $\lambda/2$ along x. Therefore the positions of the global and local minima are interchanged.

We first look for the eigenstates and eigenenergies for the motion of an atom of mass M in the periodic potentials $U_\pm(x, y)$. The energy spectrum, measured in units of the recoil energy $E_R = \hbar^2 k^2/2M$, depends only on the parameter U_0/E_R and it is the same for $U_\pm$ by symmetry. It consists of allowed energy bands, separated by forbidden gaps (fig. 1*b*)). The eigenstates are labelled $|n, \boldsymbol{q}, \varepsilon\rangle$, where the integer n is the band index, $\boldsymbol{q}$ is the Bloch vector chosen in the first Brillouin zone ($|q_x| + |q_y| \leq k$) (1) and ε stands for $g_\pm$. For the numerical calculation, we have chosen momentum states $|\boldsymbol{p}\rangle$ as basis functions, with $\boldsymbol{p} = \hbar\boldsymbol{q} + \hbar k(n_1 + n_2, n_1 - n_2)$ and $|n_i| \leq 10$ (n_i integer). The lowest levels of this spectrum can be approximated by the energy levels of a 2D harmonic oscillator located at the bottom of the well. It is also possible to identify in this spectrum the band corresponding to the «ground state» in the local minimum of $U_\pm$ (fig. 1*b*)).

In a second step, in a way similar to [3], we derive from the master equation the relaxation rates $\gamma_{i\to j}$ due to optical-pumping processes among states $|i\rangle \equiv |n, \boldsymbol{q}, \varepsilon\rangle$ and $|j\rangle \equiv |n', \boldsymbol{q}', \varepsilon'\rangle$. In order to minimize the calculations, we have taken a simplified spontaneous emission pattern. Photons are allowed to be emitted along the axes x, y and z only and their polarizations are restricted to be along these axes (2). It is then possible to keep in each band (n, ε) only a pair of states $\boldsymbol{q}_1$ and $\boldsymbol{q}_2 = \boldsymbol{q}_1 - k\boldsymbol{e}_x$, since the corresponding subset of states is closed with respect to this simplified spontaneous-emission process. To improve the accuracy of the results, we have averaged our results for two such pairs of states, with $\boldsymbol{q}_1 = k(1/4, 0)$, $q_2' = k(-3/4, 0)$ for the first pair, and $q_1' = k(1/2, 1/4)$, $\boldsymbol{q}_2 = k(-1/2, 1/4)$ for the second pair. Using the symmetries of

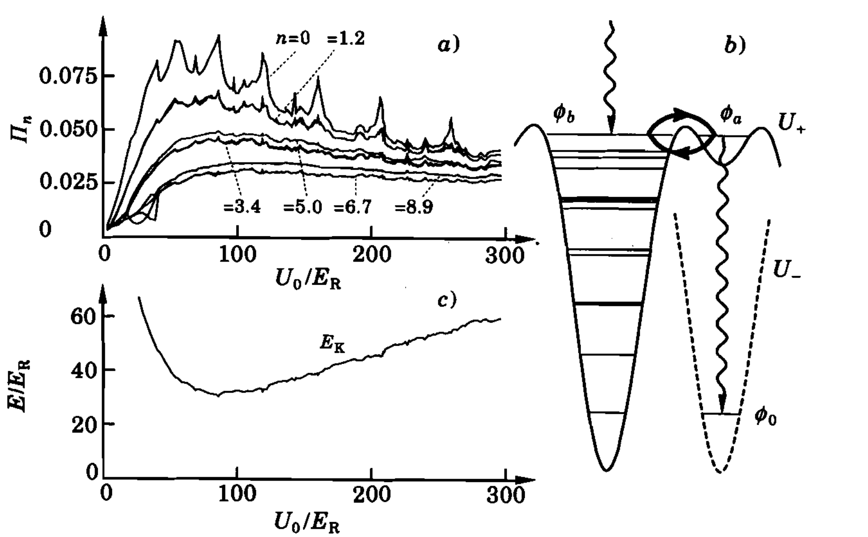

Fig. 2. – *a*) Steady-state populations Π_n of the lowest-energy bands n as functions of U_0/E_R. *b*) When the ground level in the local potential well of U_+ is close to an excited level of the main potential well, tunnelling may open an extra cooling channel towards the lowest-energy state of U_-, which enhances its steady-state population. *c*) Variations of the energy E_K with U_0/E_R.

(1) This form of the first Brillouin zone corresponds to the square lattice structure of $U_\pm(x, y)$ along the axes $(\boldsymbol{e}_x \pm \boldsymbol{e}_y)/\sqrt{2}$.

(2) In one dimension, we found for this simplified emission pattern an excess of the order of $0.2\hbar k$ in the minimum r.m.s. momentum compared to the correct one [10].

the problem ($x \leftrightarrow -x$, $y \leftrightarrow -y$, $x \leftrightarrow y$), this amounts to paving the Brillouin zone with 16 equal squares.

The steady-state populations Π_i of the various levels i are then obtained from the discrete rate equations:

$$0 = \dot{\Pi}_i = -\sum_{j \neq i} \gamma_{i \to j} \Pi_i + \sum_{j \neq i} \gamma_{j \to i} \Pi_j , \tag{2}$$

The Π_i's solving (2) characterize the steady state completely. As in 1D [3], this steady state depends on atomic variables and field parameters M, k, Γ, δ, Ω only through the parameter U_0/E_R. Indeed, each $\gamma_{i \to j}$ is a product of Γs_0 times a function of matrix elements between i and j of operators such as $\exp[\pm ikx]$ and $\exp[\pm iky]$.These matrix elements depend only on U_0/E_R, and the term Γs_0 factorizes out of the system of eqs. (2). Consequently, the Π_i's solving (2) depend only on U_0/E_R.

We have indicated in fig. 2a) the variation of the steady-state band population Π_n summed over ε, for the first ten bands, as a function of U_0/E_R. The maximum $\Pi_0 = 0.09$ is achieved for $U_0 = 86 E_R$. Figure 2a) shows several resonances for the populations of the lowest states as functions of U_0. We have checked that the main resonances appearing in fig. 2a) occur when the ground level $|\phi_a\rangle_\pm$ in the local minimum of $U_\pm$ becomes nearly degenerate with an excited bound level $|\phi_b\rangle_\pm$ in the main potential well. Tunnelling across the potential barrier separating the two wells lifts the degeneracy and produces an avoided crossing in the energy spectrum. To explain the resonances in Π_0 for these values of U_0, suppose for instance that during the Sisyphus «cascade» the atom reaches the state $|\phi_b\rangle_+$ (fig. 2b)). The coherent tunnelling coupling causes a precession of the atomic wave function towards $|\phi_a\rangle_+$. This wave function is localized in a region where the polarization of the light is σ_-, and it has a high probability of changing its internal state by an absorption–spontaneous-emission process. The ground state $|\phi_0\rangle_-$ of the main potential well of U_- is then the most probable final state for the external motion because of its strong spatial overlap with $|\phi_a\rangle_+$. The tunnelling has thus opened a channel from highly excited states of $U_\pm$ to the ground state of the main potential well of $U_\mp$. Outside of the avoided crossing, $|\phi_a\rangle_+$ is not fed anymore by tunnelling, but only by optical pumping from excited states of $U_\pm$. Because of the very different spatial dependence of $|\phi_a\rangle_+$ and these excited states, the population of $|\phi_a\rangle_+$ and the corresponding feeding of $|\phi_0\rangle_-$ are smaller.

In fig. 2c) we have plotted the variation of the average kinetic energy $E_K = \langle \boldsymbol{p}^2 \rangle / 2M$. It is minimal for $U_0 = 86 E_R$, corresponding to a «temperature» $k_B T = E_K \simeq 30 E_R$. We have also calculated the momentum distributions $\pi(\boldsymbol{p})$. They are almost isotropic, except in the far wings. Comparing fig. 2a) and 2c), we note that the resonances in the low-energy state populations appear only weakly in the variation of E_K.

We now compare our results with the ones obtained in 1D with a similar approach. Within the region where the effective «temperature» $k_B T$ is linear in $\hbar\Omega^2/|\delta|$, the slope shifts from 0.1 in 1D to 0.3 in 2D, which is closer to the experimental 3D result 0.4 [2]. A significant difference involves the maximum population of the lowest level, $\Pi_0 = 0.34$ in 1D and 0.09 in 2D. Most of this reduction is due to the degeneracy of excited levels in 2D: for a harmonic oscillator in thermal equilibrium with a frequency $\Omega_{osc} \ll k_B T/\hbar$, the population of the ground state changes from $\hbar\Omega_{osc}/k_B T$ in 1D to $(\hbar\Omega_{osc}/k_B T)^2$ in 2D. This decrease of Π_0 with increasing dimensionality is particularly important for the search of statistical effects related to quantum degeneracy. A natural «scale» for these effects is indeed a number N of atoms per potential well such that $N\Pi_0^{(3D)}$ is of the order of unity. Another important difference between 1D and 2D results concerns the steady-state spatial distributions. While this distribution was nearly flat in 1D [3], it is strongly modulated in 2D. Figure 3, obtained for $U_0 = 120 E_R$, shows a factor of 20 between its maxima and minima. Such a strongly

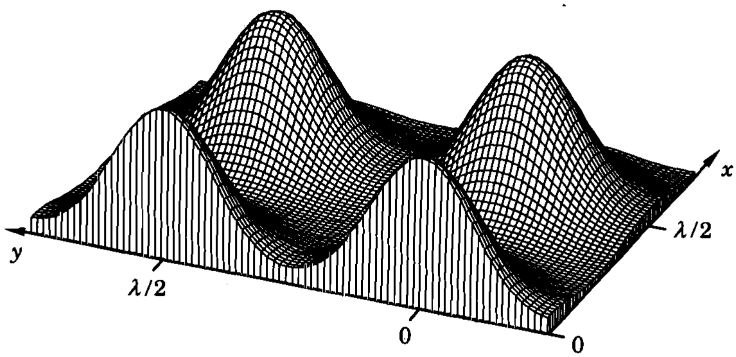

Fig. 3. – Steady-state spatial distribution for $U_0 = 120E_R$.

modulated square lattice structure should lead to an important Bragg diffraction signal of a probe laser beam.

Let us discuss the range of validity of our approach, in particular, concerning the predictions of the tunnelling-induced resonances in Π_0. These resonances can appear only if the precession between $|\phi_b\rangle_\pm$ and $|\phi_a\rangle_\pm$ has enough time to proceed in the typical delay $1/\Gamma s_0$ between two optical-pumping processes. The precession frequency is equal to the splitting between the two levels in the avoided crossing, which is found to be of the order of the recoil shift $E_R/\hbar$. We get therefore

$$\frac{E_R}{\hbar\Gamma s_0} = 4\frac{|\delta|}{\Gamma}\frac{E_R}{U_0} \gg 1 .$$

This is nothing but the standard validity condition for the secular approximation at the basis of the set of rate equations (2): the relaxation rates of the levels, typically Γs_0, must be small compared to the minimal frequency splitting between any two levels leading to a non-zero off-diagonal density matrix element in steady state. The typical value for these splittings, $E_R/\hbar$, is found either at an avoided crossing or within a group of levels which would be degenerate in a purely harmonic 2D potential, at the bottom of the wells of $U_\pm$.

For typical values of U_0 ($U_0 \sim 100E_R$), the detunings satisfying (3) are much larger than those normally used in laser-cooling experiments. Therefore, the observations of the resonances in Π_0 should require specifically designed experiments, for which alkali atoms may not be the best candidates. Indeed, the hyperfine splitting of their excited states limits the accessible detunings.

We have also studied this problem by a direct integration of the master equation giving the time evolution of the full atomic density matrix [11]. Although it is much more computationally expensive, the advantage of this second method is to be valid at any detuning. This allows one to study the transition between the large-detuning region defined by (3), for which we have recovered the predictions of the secular approach, in particular the peaks for Π_0, and the low-detuning region. In the latter case ($|\delta| < 20\Gamma$), we find, as expected, that the tunnelling-induced resonances are washed out.

In conclusion, we have presented in this letter a new feature of the quantum motion of atoms in optical molasses. The tunnelling between two excited levels of the atomic motion leads, for large detunings, to a sizable resonant variation of the population of the lowest-energy state via the opening of an extra cooling channel. We note that this type of phenomenon could also occur in 1D schemes more complex than the ones studied up to now.

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