LASER SPECTROSCOPY

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A model for an atom laser

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ABSTRACT

We present a model for an "atom laser", and we show that, under quite general conditions, such a device may lead to the accumulation of a macroscopic fraction of the atoms in a given mode of the surrounding atomic cavity. We point out the similarities between this phenomenon and the well known Bose–Einstein condensation.

Among the numerous applications of laser cooling, a most fascinating one is related to the manifestations of the statistical nature of the atoms, either bosonic or fermionic. For bosons, an actively pursued goal is the Bose–Einstein condensation; the prospects for this condensation in an atomic gas, first cooled with lasers, and further compressed by evaporative cooling, will be discussed extensively in other contributions to the present book [1]. Here we want to focus on a related, but different project: the "atom laser". In such a device we replace the photons of the standard laser by bosonic atoms, the amplification of light being replaced by the stimulated emission of atoms in a given mode of an atomic cavity. A related situation is adressed in [2], where an explicit expression is derived for the gain of a matter wave amplifier.

1. Rate equations for the atom laser

We consider here atoms with two internal levels a and b. The atoms are injected in the atom laser in state a with rate R_a. They decay to state b, and they are then trapped by an external potential forming a 3-dimensional box with a volume V (fig. 1). A photon is spontaneously emitted during the decay, with a momentum $\hbar k$. We will assume that the incident atoms in state a have a momentum of the order or smaller than $\hbar k$ to that the momenta of the atoms in b after the decay lie in a sphere of radius $p_0 = 2\,\hbar k$. Since the density of states in momentum space for a box of volume V is $V/(2\pi\hbar)^3$, the number of levels $|b, \vec{p}\rangle$ which can be reached after the decay of an incident atom is:

$$N_{\rm lev} = \frac{V}{(2\pi\hbar)^3}\frac{4\pi p_0^3}{3} = \frac{32\pi}{3}\frac{V}{\lambda^3} \qquad (1)$$

where $\lambda = 2\pi/k$. We restrict here to the case where V is much larger than λ^3. The opposite situation has been explored in [3,4].

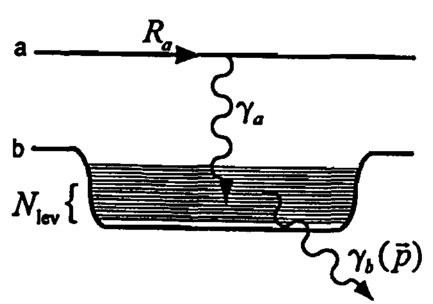

Figure 1: *A model for an atom laser: atoms are injected with rate R_a in internal state a. They decay to the internal state b, and are then trapped inside a box of volume V. The laser action originates from the stimulated emission of atoms in states $|b,\vec{p}\rangle$ when these states are already occupied.*

We now consider the evolution of the mean occupation number $n_b(\vec{p})$ of a given state $|b,\vec{p}\rangle$. We use here rate equations as it is commonly done for usual (photon) lasers [5]. The equation of motion for $n_b(\vec{p})$ consists in three terms:

$$\dot{n}_b(\vec{p}) = -\gamma_b(\vec{p})n_b(\vec{p}) + \frac{\gamma_a}{N_{lev}}N_a(1+n_b(\vec{p})) - \frac{\sigma c}{V}N_\nu n_b(\vec{p}) \qquad (2)$$

The first term describes the losses of atoms out of the cavity; we assume that the loss rate $\gamma_b(\vec{p})$ can be written as

$$\gamma_b(\vec{p}) = \gamma_{b0} + \alpha\frac{|\vec{p}|^2}{p_0^2} \qquad (3)$$

This loss rate is minimum for $\vec{p}=0$, with a quadratic increase around this value. Such a dependence can be achieved for instance using a velocity selective excitation from b to another untrapped state c [6,7].

The second term in (2) describes the feeding of the state $|b,\vec{p}\rangle$ due to the decay:

$$a \longrightarrow b + \text{photon}. \qquad (4)$$

The number of atoms in state a is N_a. When all levels $|b,\vec{p}\rangle$ are empty, the total decay rate of state a is γ_a and we assume for simplicity that the decay rates towards each of the N_{lev} accessible levels $|b,\vec{p}\rangle$ have the same value γ_a/N_{lev}. When level $|b,\vec{p}\rangle$ is occupied, both the spontaneous and the stimulated emission of a bosonic atom in $|b,\vec{p}\rangle$ are included in (2), leading to the factor $1+n_b(\vec{p})$.

The last term of (2) corresponds to the reverse process

$$b + \text{photon} \longrightarrow a. \qquad (5)$$

i.e. to the reabsorption of emitted photons. σ is the absorption cross-section of a photon, and N_ν denotes the number of photons present in the volume V.

The equations of evolution for the number of atoms in state a, N_a, and the number of photons N_ν are given by:

$$\dot{N}_a = R_a - \gamma_a N_a\left(1+\frac{N_b}{N_{lev}}\right) + \frac{\sigma c}{V}N_\nu N_b \qquad (6)$$

$$\dot{N}_\nu = -\gamma_\nu N_\nu + \gamma_a N_a\left(1+\frac{N_b}{N_{lev}}\right) - \frac{\sigma c}{V}N_\nu N_b \qquad (7)$$

We have put $N_b = \sum_{\vec{p}} n_b(\vec{p})$, and $\gamma_\nu{}^{-1}$ denotes the time of flight of a photon across the box of volume V ($\gamma_\nu \sim c/V^{1/3}$); we assume that all photons escape when they reach the border of the atomic cavity.

2. The steady-state of the atom-laser

From the three equations of evolution of $n_b(\vec{p}), N_a, N_\nu$, we can now determine the steady-state of the atom laser. After some algebra, we get:

$$n_b(\vec{p}) = \frac{1+fu}{\frac{1+f}{r_a}\gamma_b(\vec{p}) - 1 + u} \qquad (8)$$

where we have put $f = N_b/N_{lev}$, $r_a = R_a/N_{lev}$ (feeding rate per mode) and $u = (\sigma c N_{lev})/(\gamma_\nu V)$. The determination of the steady-state requires the determination of f, which is done by summing (8) over $\vec{p}$ and by solving the resulting equation.

In order to get a macroscopic accumulation of population in the single quantum state with minimal losses $|b,\vec{p}=0\rangle$, a necessary condition is $u<1$, so that f can approach the value for which $(1+f)\gamma_{b0}/r_a = 1-u$ and $|b,\vec{p}=0\rangle$ can be macroscopically populated. One can give a simple physical interpretation of this condition $u<1$. Suppose that one atom is sitting in $|b,\vec{p}=0\rangle$ and that a second atom is brought in state a. The final possible states are either the two atoms in $|b,\vec{p}=0\rangle$, which corresponds to a gain for our "laser", no atom in $|b,\vec{p}=0\rangle$, which corresponds to a loss, or one atom in $|b,\vec{p}=0\rangle$ and another in $|b,\vec{p}\neq 0\rangle$, which we can ignore. The gain probability taking into account both the spontaneous and stimulated processes is $\mathcal{P}_+ = 2/N_{lev}$. The loss occurs when the photon emitted in the decay of the second atom from a to b is reabsorbed by the atom initially in $|b,\vec{p}=0\rangle$; the corresponding probability is $\mathcal{P}_- \simeq \sigma V^{-2/3}$. The intuitive necessary condition for "single mode lasing" $\mathcal{P}_+ > \mathcal{P}_-$ is nothing but $u>1$, within a numerical factor.

In the remaining part of this section, we assume that $\sigma=0$, so that $u=0$. This simplifying assumption is strictly speaking non physical, but we will see later that it can be justified for some realistic situations. The steady-state population of $|b,\vec{p}\rangle$ is then

$$n_b(\vec{p}) = \frac{1}{\frac{1+f}{r_a}\gamma_b(\vec{p}) - 1} \qquad (9)$$

which has a mathematical structure very close to the standard Bose-Einstein distribution. In particular we choose here a quadratic variation of $\gamma_b(\vec{p})$ around $\vec{p}=0$, as it is the case for the factor $\exp(p^2/2Mk_BT)$ entering into the Bose–Einstein distribution. It is therefore quite intuitive that under certain conditions upon the "intensive variables" r_a, γ_{b0}, α, we will find that the level $|b, \vec{p} = 0\rangle$, for which losses are minimal, may be macroscopically occupated. The remaining of the discussion is actually analogous to the textbook derivation of the Bose–Einstein condensation [8].

The equation determining f is

$$f = \frac{1}{N_{\rm lev}} \sum_{\vec{p}} \frac{1}{\frac{1+f}{r_a}\gamma_b(\vec{p}) - 1} \qquad (10)$$

A similar set of equations has been written by Casperson for the case of a photonic multimode laser [9]. When going to the continuous limit (box size $V^{1/3}$ large compared to λ), (10) becomes:

$$f = \frac{3}{4\pi p_0^3} \int_{|\vec{p}|<p_0} \frac{d^3p}{\frac{1+f}{r_a}\gamma_b(\vec{p}) - 1} = \frac{3r_a}{\alpha(1+f)} \left(1 - \frac{\arctan g}{g}\right) \qquad (11)$$

where

$$g = \sqrt{\frac{\alpha(1+f)}{\gamma_{b0}(1+f) - r_a}} \qquad (12)$$

From a graphical analysis, we check that the transcendental equation (11) has a solution only if

$$\frac{r_a}{\gamma_{b0}} - 1 \leq 3\frac{\gamma_{b0}}{\alpha} \qquad (13)$$

Suppose first that this condition is valid; we determine f using (11) and we can then deduce the population of each level $|b, \vec{p}\rangle$ using (8). No singularity appears for this set of populations, each of them going to 0 when the volume V is increased while the three "intensive variables" r_a, γ_{b0}, α are kept constant. We note that in this region the value of f solving (11) is always larger than the value $f_{\min} = (r_a/\gamma_{b0}) - 1$ for which the parameter g becomes infinite.

Consider now a situation where the condition (13) is violated. As for the Bose condensation situation, the continuous limit can only be taken after treating separately the most populated state $\vec{p} = 0$:

$$N_b = n_b(\vec{0}) + \sum_{\vec{p}\neq 0} n_b(\vec{p}) = n_b(\vec{0}) + \frac{V}{(2\pi\hbar)^3} \int d^3p \; n_b(\vec{p}) \qquad (14)$$

The value of the parameter f solution of this equation is very close[1] to $f_{\min}$ so that the number of atoms N_b' in states $|b, \vec{p}\rangle$ with $\vec{p} \neq 0$ can be evaluated taking $f = f_{\min}$:

$$N_b' = \sum_{\vec{p}\neq 0} n_b(\vec{p}) = \frac{V}{(2\pi\hbar)^3} \int d^3p \; n_b(\vec{p}) = 3N_{\rm lev}\frac{\gamma_{b0}}{\alpha} \qquad (15)$$

[1]This is very similar to the fact that the chemical potential is very close to 0 when the Bose condensation threshold is reached.

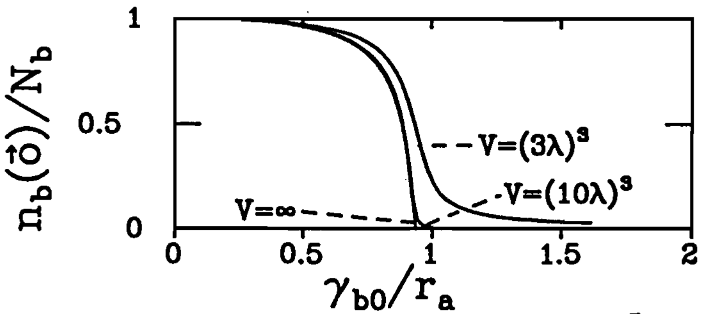

Figure 2: *Variations, for various sizes of the box volume V, of $n_b(\vec{0})/N_b$ as a function of γ_{b0}/r_a, with $\alpha = 40\,r_a$. The density at threshold is $N_b/V \simeq 3/\lambda^3$.*

The total number of atoms in the b state is given by:

$$N_b = f N_{\rm lev} = N_{\rm lev}\left(\frac{r_a}{\gamma_{b0}} - 1\right), \qquad (16)$$

The difference between N_b and N_b' gives the population of the single quantum state $|b, \vec{p} = 0\rangle$ (see fig.2). As for the Bose condensation, this population is macroscopic, in the sense that the "condensate" density $n_b(\vec{0})/V$ tends to a finite value when the volume V goes to infinity, r_a, γ_{b0}, α beeing kept constant.

3. Discussion

We have presented here an open model for an atom laser, where atoms are injected in an atomic cavity in state a and where they leave the cavity by being transferred with a rate $\gamma_b(\vec{p})$ from state b to an untrapped state c. One can also consider a closed model where atoms leaving b are "recycled" in a. The only modification in our initial set of equations is to replace R_a in (6) by $\sum_{\vec{p}} \gamma_b(\vec{p}) n_b(\vec{p})$. One obtains for such a closed model results similar to the present ones; in particular, one recovers the existence of a threshold at which the single quantum state $|b, \vec{p}\rangle$ gets macroscopically populated.

To implement such a scheme experimentally, a possible way is to choose for a and b two hyperfine ground states of an alkali atom. Atoms are prepared in a from a magneto-optical trap, which can lead to a rate $r_a \sim 1$ atom/mode/second. The decay from a to b occurs *via* the absorption of a photon of an additional depumping laser and the spontaneous emission of a Raman photon. At this stage, one has to examine carefully the hypothesis $\sigma \sim 0$ used here. Actually, if the additional laser is purely monochromatic, the re-absorption cross section of the spontaneous Raman photon is large, of the order of λ^2; because of stimulated Raman processes, this large cross section is found even if the depumping laser is detuned far from any atomic resonance [10]. We propose to use a broad-band depumping laser, with a detuning δ from an atomic resonance and with a width $\Delta\omega$ such that $\gamma_a \ll \Delta\omega \ll \delta$; in this case the stimulated contribution to the cross-section is reduced by a factor $\Delta\omega/\gamma_a$ [11].

Let us point out some differences between this atom-laser scheme and Bose–Einstein condensation. First, it is clear that the population distribution obtained in (9) is not a thermal equilibrium. We did not consider here any thermostat which would impose a temperature to the atomic gas. Second, we do not rely on elastic collisions to build up the condensate. This condensate appears thanks to the decay process (4) involving a stimulated emission of atoms in an already occupied state.

Finally we note that this "condensation" scheme can be extended to systems with lower dimensionality provided one can master precisely the loss rate $\gamma_b(\vec{p})$. In the present discussion, the loss rate varies quadratically with $|\vec{p}|$. This ensures that the 3D integral in (11) converges when $f = f_{\min}$, and it allows a macroscopic population to appear in $|b, \vec{p} = 0\rangle$. In a 2D system, the integral in (11) would diverge when f goes to $f_{\min}$ so that the equation equivalent to (11) would have a solution for any set of parameters r_a, γ_{b0}, α and no condensation would be found. The situation is different for a linear dependence of the loss rate in $|\vec{p}|$. In this case, the integral in (11) will converge in $f = f_{\min}$ both for a 3D and a 2D system. The condensation phenomenon could then be observed also in 2D, which the practical advantage of a faster elimination than in 3D of the emitted photons.

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