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Practical scheme for a light-induced gauge field in an atomic Bose gas

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We propose a scheme to generate an Abelian gauge field in an atomic gas using two crossed laser beams. If the internal atomic state follows adiabatically the eigenstates of the atom-laser interaction, Berry's phase gives rise to a vector potential that can nucleate vortices in a Bose gas. The present scheme operates even for a large detuning with respect to the atomic resonance, making it applicable to alkali-metal atoms without significant heating due to spontaneous emission. We test the validity of the adiabatic approximation by integrating the set of coupled Gross-Pitaevskii equations associated with the various internal atomic states, and we show that the steady state of the interacting gas indeed exhibits a vortex lattice, as expected from the adiabatic gauge field.

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A major motivation of current research with atomic quantum gases is the prospect of simulating the physics of condensed matter systems (see, for example, [1]). One objective is to gain new insight into strongly correlated states of matter such as the fractional quantum Hall effect exhibited by electrons in a magnetic field [2]. To study such systems with cold atoms, one needs to mimic a gauge field with which the neutral atoms interact as if they were charged. A well-known method consists in setting the gas into rotation and transforming to the corotating reference frame, where the physics is governed by the same Hamiltonian that describes charged particles in a magnetic field. At the mean-field level the corresponding gauge field leads to the nucleation of vortices, which has been observed experimentally (see [3] and references therein). However, technical difficulties have so far prevented the strongly correlated regime from being reached, and it is worthwhile to contemplate alternative approaches.

A promising possibility is the use of geometric potentials induced by laser fields. To comprehend the underlying mechanism, consider atoms with a ground state g, which is degenerate in the absence of any external perturbation (angular momentum $J_g>0$). Neglecting in a first step the population of excited levels, the atomic energy eigenstates $|\psi_\alpha\rangle$ ($\alpha=0,\dots,2J_g$) in the presence of the laser fields are linear combinations of the Zeeman substates $|g,J_g,m\rangle$ ($m=-J_g,\dots,J_g$). Both the eigenenergies E_α and the states $|\psi_\alpha\rangle$ depend on position via the spatial variation of the light fields. If the center-of-mass (c.m.) motion is slow enough, an atom initially prepared in one of these internal states, say $|\psi_0\rangle$, will remain in this state and its c.m. wave function will acquire a geometric phase [4]. This so-called Berry phase [5] corresponds to a gauge field $\{\boldsymbol{A},U\}$ appearing in the effective Hamiltonian of the c.m. motion:

$$H=\frac{[\boldsymbol{p}-q\boldsymbol{A}(\boldsymbol{r})]^2}{2M}+E_0(\boldsymbol{r})+U(\boldsymbol{r}), \quad (1)$$

where M is the atomic mass,

$$\boldsymbol{A}=i\hbar\langle\psi_0|\,\boldsymbol{\nabla}\,(|\psi_0\rangle), \qquad U=\frac{\hbar^2}{2M}\sum_{\alpha\neq 0}|\langle\psi_0|\,\boldsymbol{\nabla}\,(|\psi_\alpha\rangle)|^2, \quad (2)$$

and with the charge $q=1$. Several configurations to create gauge fields with laser beams have been proposed and analyzed [6–14], and experimental evidence of these potentials has been provided in the context of cold atomic gases [15,16]. In particular, the proposal of [13] uses two counterpropagating beams driving the two transitions of an atom with a Λ level scheme [Fig. 1(a)]. Such a configuration emerges when the two beams are circularly polarized with positive and negative helicity, respectively, and both the ground (g) and the excited (e) states have unit angular momentum ($J_g=J_e=1$) [17]. In this case $|\psi_0\rangle$ can be chosen as the "dark" superposition state of $|g,J_g,m=\pm 1\rangle$. Unfortunately, this scheme is not applicable to the commonly used alkali-metal atoms even though they may have some hyperfine states with the proper angular momenta. The hyperfine splitting of the relevant excited manifold being typically smaller than 100 linewidths, nonresonant coupling to other hyperfine states with $J_e\neq 1$ will destabilize the superposition state, and heating due to spontaneous emission will eventually mask the effect searched for.

In this Rapid Communication we first propose a simple planar scheme which overcomes the above-mentioned handicap. It incorporates two crossed laser beams whose frequencies are far detuned from the atomic resonance transition, which reduces the rate of spontaneous emission processes to a negligible value. Our scheme produces an effective gauge field that varies smoothly over the area of the atomic cloud

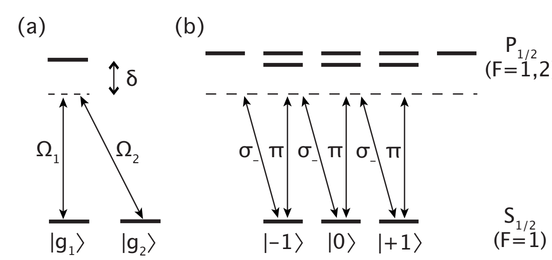

FIG. 1. (a) Λ-type system used for a simple modeling of the atom-laser coupling. (b) Atomic scheme relevant for a 3/2 nuclear spin, as for ^{87}Rb or ^{23}Na.

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and allows for the observation of several vortices in a cold interacting spinor Bose gas. In a second step, we determine the ground state of the gas by solving numerically the coupled set of Gross-Pitaevskii equations (GPEs) associated with the various internal (ground and excited) atomic states. We show that the ground state of the system indeed contains several vortices with the predicted surface density. In contrast to previous approaches [18] where the gauge fields (2) were explicitly included in the Hamiltonian, we do not *a priori* assume the adiabatic limit to be valid. Hence, our study can be considered as a validity check of this approximation for an interacting gas. In the last part of this paper we investigate whether the vortices can be nucleated by slowly turning on the geometric light potential.

The necessity of using off-resonant lasers strongly limits the choice of schemes that lead to a nontrivial gauge field. To be explicit, we first remark that a frequency detuning large compared to the hyperfine splitting of the excited manifold causes the nuclear spin to become irrelevant for the description of the excited state. Consider, for example, the D_1 transition between the states $S_{1/2}$ and $P_{1/2}$ of an alkali-metal atom, which is sketched in Fig. 1(b) for the case of a nuclear spin 3/2, as for ^{87}Rb or ^{23}Na. Since only the electronic angular momentum is relevant for evaluating the matrix elements of the atom-laser coupling, the energy level structure can be regarded as a simple $J_g=1/2 \leftrightarrow J_e=1/2$ system. At the same time this restricts the polarizations of the laser beams that can be applied to create the internal superposition state: A spin flip from $m=-1/2$ to $m=+1/2$, for instance, corresponds to an angular momentum transfer of $+\hbar$, so a two-photon transition requires light with circular (σ_- or σ_+) as well as linear (π) polarization. Such a configuration cannot be achieved with counterpropagating waves as in [13] for which the change $\Delta m = 0, \pm 2$.[1] Note that it is important that the detuning is not large compared to the *fine* structure splitting between the $P_{1/2}$ and $P_{3/2}$ manifolds. Otherwise the electronic spin S loses its significance, and one is left with an effective $J_g=0 \leftrightarrow J_e=1$ transition where no coherent level superposition is generated by the atom-light interaction.

For the sake of simplicity, we first treat the case of a three-level Λ system. The transposition to a realistic internal level structure will be discussed later on. The two ground states $|g_1\rangle$ and $|g_2\rangle$ are coupled to the excited state by laser fields with spatially varying Rabi frequencies $\Omega_1(\boldsymbol{r})$ and $\Omega_2(\boldsymbol{r})$, respectively [Fig. 1(a)]. The Hamiltonian for an atom in the light field at any point $\boldsymbol{r}$ reads

$$H = \frac{\boldsymbol{p}^2}{2M} + V(\boldsymbol{r}) + H_{\mathrm{AL}}(\boldsymbol{r}). \tag{3}$$

The atom is confined in a two-dimensional trapping potential $V(\boldsymbol{r}) = (\omega_x^2 x^2 + \omega_y^2 y^2)/2M$. The part H_{AL} of the Hamiltonian acts on the internal degrees of freedom only and describes

[1]Counterpropagating beams can induce a $\Delta m = \pm 1$ transition if a comparatively large magnetic field is applied and the frequencies of the two beams are properly shifted. Yet, the nonlinear Zeeman effect may in that case lead to spurious exothermic spin-changing collisions.

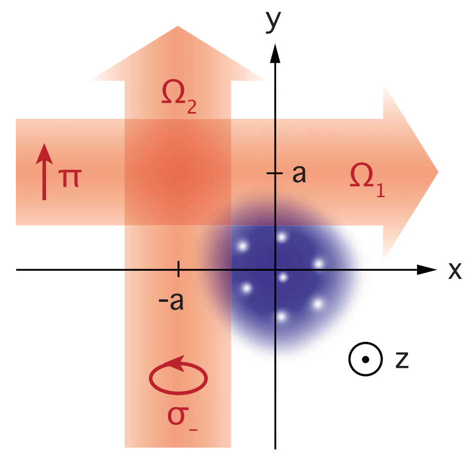

FIG. 2. (Color online) Planar scheme to create a gauge field for atoms with two crossed far-detuned laser beams which are displaced by a distance $\pm a$ from the center position of the atomic cloud.

the atom-laser coupling. Using the rotating wave approximation, it can be written as

$$H_{\mathrm{AL}} = \hbar \begin{pmatrix} 0 & 0 & \Omega_1/2 \\ 0 & 0 & \Omega_2/2 \\ \Omega_1^*/2 & \Omega_2^*/2 & -\delta \end{pmatrix}, \tag{4}$$

where $\delta = \omega_{\mathrm{L}} - \omega_{\mathrm{A}}$ is the detuning of the laser frequency ω_{L} with respect to the atomic resonance frequency ω_{A}. We suppose that the atom is prepared in the eigenstate

$$|\psi_0(\boldsymbol{r})\rangle = \cos(\theta/2)|g_1\rangle + e^{-i\phi}\sin(\theta/2)|g_2\rangle \tag{5}$$

of H_{AL}, where we have set $\cos\theta = (|\Omega_1|^2 - |\Omega_2|^2)/\Omega^2$, $e^{i\phi}\sin\theta = -2\Omega_1^*\Omega_2/\Omega^2$, and $\Omega = (|\Omega_1|^2 + |\Omega_2|^2)^{1/2}$. Here $|\psi_0\rangle$ is a noncoupled (dark) state, separated from the next eigenstate by an energy given by $\varepsilon = \hbar\Omega^2/4\delta$ in the low-intensity limit $\Omega \ll |\delta|$. The general expressions (2) for the gauge potentials yield in this case

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\hbar}{2}(1 - \cos\theta)\,\boldsymbol{\nabla}\phi, \tag{6}$$

$$U(\boldsymbol{r}) = \frac{\hbar^2}{8M}[(\boldsymbol{\nabla}\theta)^2 + \sin^2\theta\,(\boldsymbol{\nabla}\phi)^2]. \tag{7}$$

We now introduce a simple implementation scheme to generate a gauge field with two far-detuned Gaussian laser beams. As the beams will in practice need to be σ_- and π polarized (or equivalently σ_+ and π), we choose a crossed-beam configuration. Our beams are displaced by a distance $\pm a$ (of the order of the beam waists) from the x and the y axis, respectively, as shown in Fig. 2, in order to obtain a nonvanishing magnetic field at the origin. The Rabi frequencies thus read $\Omega_1(\boldsymbol{r}) = \Omega_0 \exp[ikx - (y-a)^2/w^2]$ and $\Omega_2(\boldsymbol{r}) = \Omega_0 \exp[iky - (x+a)^2/w^2]$, where w denotes the waist of the beams and k the wave vector. Expanding the elements of the matrix (4) up to second order around the origin and plugging the results into Eq. (6), we obtain the vector potential $\boldsymbol{A}(\boldsymbol{r}) = \hbar k(\boldsymbol{e}_y - \boldsymbol{e}_x)[1/2 - a(x+y)/w^2]$, yielding the effective magnetic field to lowest order:

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{\nabla} \times \boldsymbol{A} = \frac{2\hbar k a}{w^2}\boldsymbol{e}_z. \tag{8}$$

An important quantity characterizing the effect of the magnetic field is the cyclotron frequency $\omega_c = q|\boldsymbol{B}|/M$, with $q=1$ in the present case. For our configuration we get $\omega_c = 2\hbar k a/Mw^2$ close to the origin. For comparison, in a gas rotating with angular frequency Ω_{rot} one has $\omega_c = 2\Omega_{\mathrm{rot}}$. In the situation where several vortices are present in the ground state, the surface density of vortices is related to ω_c by [3]

$$\rho_v = \frac{M\omega_c}{2\pi\hbar} = \frac{ka}{\pi w^2}. \tag{9}$$

The typical radius of the atom cloud can be chosen such that $R \sim a \sim w$ so that the number of vortices inside the cloud is $\pi R^2 \rho_v \sim ka$. Owing to the fast phase variation $\sim k \gg 1/a$ of the light field we may thus expect a significant number of vortices to be present in a steady state cloud. This number is ultimately limited by the higher-order terms in the expansion of the gauge field. If the displacement a is chosen too large compared to the waist w, the magnetic field will be inhomogeneous and the scalar potential anharmonic.

To obtain the scalar potential, we note that the first term in expression (7) is small compared to the second one $\sim(\boldsymbol{\nabla}\phi)^2 = 2k^2$ and can therefore be neglected. An expansion around the origin up to second order then leads to

$$U(\boldsymbol{r}) = \frac{\hbar^2 k^2}{4M} - \frac{1}{2}M\omega_c^2\left(\frac{x+y}{\sqrt{2}}\right)^2, \tag{10}$$

which reduces the trapping frequency along the axis $x=y$. This anisotropy can be compensated for by adapting the initial trapping potential V.

Up to now our scheme (like other previous proposals) relies on the assumption that the atomic motion is slow enough so that the atoms follow adiabatically the state $|\psi_0(\boldsymbol{r})\rangle$. Although reasonable, this approximation could be questioned in the vicinity of a vortex core, where the velocity field can become arbitrarily large. To settle this point we have numerically determined the ground state of a trapped gas of interacting bosonic atoms with a Λ-type internal structure irradiated by lasers in the configuration of Fig. 2. The gas is assumed to move only in the xy plane, and the third motional degree of freedom is assumed to be frozen to the ground state of a strongly confining potential along the z direction (frequency ω_z). Spontaneous emission processes are included by adding the complex term $-i\hbar\Gamma$ to the energy of the excited state. For simplicity we take the same s-wave scattering length a_s for all internal states and hence disregard spin-spin interactions which could lead to spin exchange collisions. This approximation is reasonable for the hyperfine ground states of ^{87}Rb but not necessarily for other atomic species. For the excited state it should in general be unproblematic as the atoms spend their time primarily in the ground state manifold. The interactions are treated at the mean-field level, and the ground state of the system is obtained using an evolution in imaginary time of the three coupled GPEs associated with the three internal atomic states. A typical result is shown in Fig. 3(a). We find that the atoms accumulate as expected in the noncoupled internal state $|\psi_0(\boldsymbol{r})\rangle$ while the population of states other than $|\psi_0\rangle$ is evanescent (fraction below 2×10^{-5} for the parameters of Fig. 3). Furthermore, the equilibrium c.m. distribution indeed exhibits a vortex lattice whose vortex density at the center is in good agreement with the prediction of Eq. (9). The slight asymmetry of the cloud shape stems from nonharmonic contributions to the scalar potential (7).

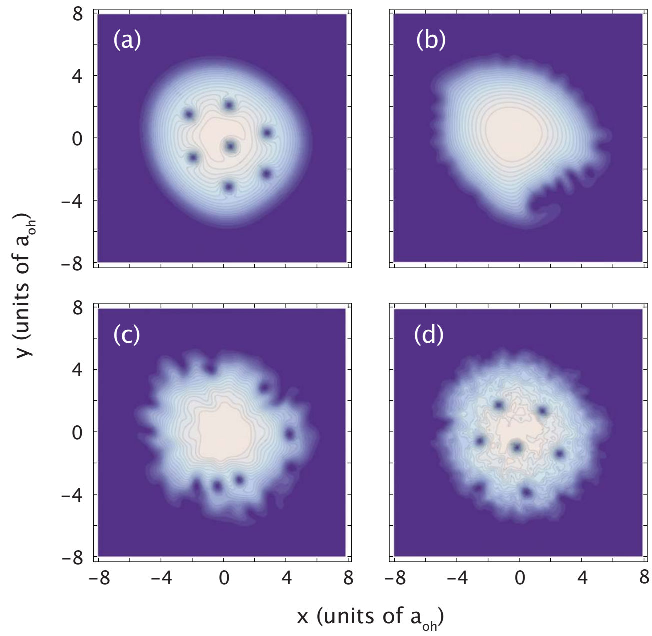

FIG. 3. (Color online) Density distributions of a two-dimensional (2D) gas of bosonic atoms with a Λ level scheme and rubidiumlike parameters: resonance wavelength 795 nm, $M=1.45\times10^{-25}$ kg, natural linewidth of the excited state $\Gamma/2\pi=6$ MHz. The harmonic trapping potential in the xy plane, including the scalar potential U, is isotropic with a trapping frequency $\omega/2\pi = 40$ Hz and a ground state extension $a_{\mathrm{ho}}=(\hbar/M\omega)^{1/2}=1.7\ \mu$m. Atomic interactions are chosen such that $G=Na_s(8\pi M\omega_z/\hbar)^{1/2}=800$. The laser configuration is as sketched in Fig. 2, with $a=w=25a_{\mathrm{ho}}$, $\Omega_0/\Gamma=280$, and $\delta/\Gamma=4\times10^5$. (a) Steady state distribution. (b)–(d) Distributions obtained after ramp durations of the second laser beam of (b) $50\omega^{-1}$, (c) $200\omega^{-1}$, and (d) $1000\omega^{-1}$.

Vortex nucleation following the sudden application of a gauge field usually occurs through a turbulent phase which lasts a fraction of a second, followed by a relaxation to the ground state via a dissipative process [19–21]. Another possible route could be a smooth evolution from a no-vortex state to a state with vortices as the gauge field is slowly turned on. Usually this process is forbidden, at least at the mean-field level, because of the different parities of the initial and final states. However, the gauge field generated in the present scheme does not have any definite parity property and such a smooth evolution should in principle be possible. To test whether it can be implemented in practice, we assume that one laser beam is switched on first in order to lift the degeneracy between the two ground state levels. Once it has reached its full intensity, we ramp up linearly the intensity of the second laser beam in an adjustable time T. Starting with a trapped gas in its ground state in the presence of one beam (a Bose-Einstein condensate without vortices), we propagate the wave function in real time using the GPEs. The resulting density distributions are shown in Fig. 3(bcd). For short ramp times $T \lesssim 200\omega^{-1}$ many vortices are situated around the border of the cloud but they have barely moved toward the center [see Figs. 3(b) and 3(c)]. For slower ramps [Fig. 3(d)] the final state looks closer to the expected ground state shown in Fig. 3(a), although the convergence is not perfect. We conclude that for these parameters the time scale for a smooth turn-on of the geometric potential must be longer than $1000\omega^{-1}$ (≈ 4 s for $\omega/2\pi = 40$ Hz), which seems too long for a practical implementation. Probably vortex nucleation in these geometric potentials will be more easily achieved by following the same route as for a rotating trap, i.e., by suddenly applying the gauge field to a condensate at rest and by letting the cloud evolve to its new steady state through a transient turbulent phase [19–21].

The analytical derivation presented above for a three-level system can be straightforwardly extended to the case of a realistic atomic transition. Although the results remain qualitatively unchanged, it is instructive to identify some minor differences. First, unlike for a Λ system, the eigenstates of the atom-laser coupling are not "dark" in the sense that they all contain a nonzero admixture of the excited level. However, the residual photon scattering rate can be made very small. The parameters given in the caption of Fig. 3 lead to a scattering rate of 0.15 photons/s per atom, which is negligible on the time scale of a typical experiment. Second, we note that the level scheme of Fig. 1(b) leads to some modifications of the geometric gauge field with respect to the Λ system. After optimization of the relative intensities of the σ_-- and π-polarized laser beams, one obtains that the effective magnetic field at the origin is multiplied by the factor $8J_g/(3\sqrt{3})$ with respect to the result (8). For the $J_g = 1$ ground state of ^{87}Rb, for example, this leads to a 50% increase of the vortex density at the trap center.

In conclusion, we have proposed in this Rapid Communication a scheme to create a gauge field in a cold atomic gas with negligible heating due to photon scattering. We have also validated the adiabatic approximation from which this gauge field emerges by solving the set of coupled Gross-Pitaevskii equations associated with the atomic internal levels. In particular, the ground state of a trapped 2D Bose gas indeed contains several regularly arranged vortices, with the density predicted from the gauge field in the adiabatic approximation. The feasibility of our simple scheme should render possible an experimental observation of a light-induced magnetic field.

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