


Emergence of a Landau Level Structure in Dark Optical Lattices

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An optical flux lattice is a set of light beams that couple different internal states of an atom, thereby producing topological energy bands. Here we present a configuration in which the atoms exhibit a dark state, i.e., an internal state that is not coupled to the light. At large light intensity, the low-energy dynamics is restricted to the dark state, leading to an effective continuum model with a Landau-level-like structure. This structure is dramatically different from that of usual topological optical lattices, which lead to discrete models in the tight-binding limit. For well-chosen atomic species, the proposed system is essentially immune to heating due to photon scattering, making it a highly promising way to emulate the integer or fractional quantum Hall effect.

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The generation of topological bands for atoms or photons moving in a periodic potential has been recently the subject of many theoretical and experimental studies (for reviews, see, e.g., Refs. [1,2]). It opens the way to a simulation of the quantum Hall effect, from the integer to the fractional regimes when the role of interactions increases. Among the various schemes that have been considered for atoms, optical flux lattices (OFLs) involve laser fields inducing a spatially periodic coupling between different internal states of the atom. Suitable laser configurations give rise to topological Bloch bands, with a periodic effective magnetic flux allowing cold atoms to experience conditions analogous to those found in quantum Hall states. OFLs have the advantage of not requiring any time-modulated parameters (which may lead to undesired heating), while providing a large flux density, typically one or two flux quanta per unit cell [3]. However, OFL schemes proposed so far, when they are aimed at atomic species such as alkali-metal atoms, may induce significant photon scattering, which limits the available time during which the atomic gas remains cold enough to observe topology-induced effects [4,5].

In this Letter, we propose a modified version of the OFL concept that operates with a so-called “dark state,” thus reducing considerably the heating problem due to photon scattering. Dark-state lattices [6,7] have recently been identified as a method for achieving subwavelength structures, either in one [8–10] or two dimensions [11,12]. Here we propose a scheme providing an energy spectrum remarkably close to the Landau level structure for a charged particle in a uniform magnetic field. For realistic

parameters, we obtain a series of many equidistant, non-overlapping, topological bands, each with a Chern index equal to 1. Being restricted to the dark internal state, the atoms remain weakly coupled to light, giving rise to delocalized orbitals. This behavior contrasts with other realizations of topological bands in optical lattices, which can be described by discrete lattice models such as the Haldane and Harper-Hofstadter models [13–15]. As an illustration of the topological character of the bands, we check that an ideal Fermi gas populating the ground band should exhibit an incompressible bulk surrounded by chiral edge states, and weakly interacting bosons should organize themselves into an ordered vortex lattice.

The atom-laser coupling—We consider atoms for which a Λ -type optical transition can be isolated, with two states $|g_{\pm}\rangle$ from the ground-level manifold and a state $|e\rangle$ from an excited manifold [see Fig. 1(a)]. The OFL is formed by two sets of monochromatic laser beams, with set 1 (respectively, 2) at frequency ω_1 (respectively, ω_2) close to the frequency ω_+ of the $|g_+\rangle \leftrightarrow |e\rangle$ transition (respectively, ω_- for $|g_-\rangle \leftrightarrow |e\rangle$) [see Fig. 1(b)]. We assume that these frequencies fulfill the Raman resonance condition $\omega_1 - \omega_2 = \omega_+ - \omega_-$ so that one can identify a space-dependent dark-state, linear combination of $|g_{\pm}\rangle$ [7].

The atom-laser coupling reads in the $\{|g_+\rangle, |g_-\rangle\}$ basis,

$$\hat{V}(\mathbf{r}) = V_0 \begin{pmatrix} |\alpha_+|^2 & \alpha_+^* \alpha_- \\ \alpha_+ \alpha_-^* & |\alpha_-|^2 \end{pmatrix}, \quad (1)$$

where V_0 is a real, positive energy and $\alpha_{\pm}(\mathbf{r})$ dimensionless complex amplitudes. Here we adiabatically eliminated the excited state $|e\rangle$ assuming that the detuning from resonance $\Delta \equiv \omega_1 - \omega_+ = \omega_2 - \omega_-$ is positive and large compared to the Rabi frequency characterizing the atom-laser coupling (see Appendix A for details) [16].

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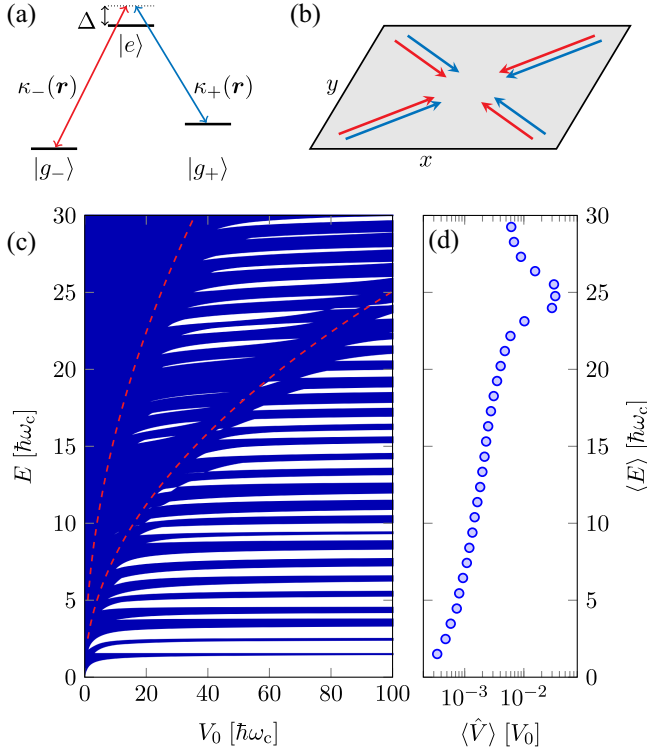


FIG. 1. (a) The Λ level scheme considered in this Letter, involving light fields at two close frequencies (red and blue arrows). (b) Spatial configuration of the lasers producing the dark optical lattice, with two standing waves along the directions $x \pm y$ for each frequency. (c) Band spectrum of the flux lattice as a function of the light coupling amplitude V_0 . In the regime $V_0 \gg \hbar\omega_c$, we find at low energy a succession of narrow energy bands with dominant dark-state character and an almost uniform spacing $\simeq \hbar\omega_c$. The red lines give a rough estimate of the first two bands with dominant bright-state character. They correspond to the energy levels $(n+1)\hbar\Omega_{\text{bright}}$ (with n integer) in the lattice potential experienced by the bright state in the large depth limit of independent harmonic wells. (d) Mean value $\langle \hat{V} \rangle$ of the light shift as a function of the mean total energy $\langle E \rangle$, averaged over the quasimomentum \mathbf{q} , for the successive bands. The small value of the light shift for the low-energy bands shows that they predominantly populate the dark state.

The dark state $|\mathcal{D}(\mathbf{r})\rangle \propto \alpha_-(\mathbf{r})|g_+\rangle - \alpha_+(\mathbf{r})|g_-\rangle$ is the eigenstate of \hat{V} with energy 0. The other eigenstate of \hat{V} is the bright state $|\mathcal{B}(\mathbf{r})\rangle \propto \alpha_+(\mathbf{r})|g_+\rangle + \alpha_-(\mathbf{r})|g_-\rangle$ with energy $E_{\text{bright}}(\mathbf{r}) = V_0(|\alpha_+(\mathbf{r})|^2 + |\alpha_-(\mathbf{r})|^2)$. This energy is positive everywhere so that the dark state is always the local ground state of $\hat{V}(\mathbf{r})$.

In this Letter, we will choose the following expressions for the coefficients $\alpha_{\pm}(\mathbf{r})$ [17]:

$$\alpha_+(\mathbf{r}) = \sin X + i \sin Y, \quad (2)$$

$$\alpha_-(\mathbf{r}) = \cos X + \cos Y, \quad (3)$$

where we set $X = k(x+y)/2$ and $Y = k(y-x)/2$. Here we assumed that the two wave numbers of the beams

forming the lattice, $k_i = \omega_i/c$ ($i = 1, 2$), are close to each other and we set $k\sqrt{2} \equiv k_1 \approx k_2$. The light-shift coupling \hat{V} is then invariant upon discrete translations by $d\mathbf{e}_x$ and $d\mathbf{e}_y$, where $d = 2\pi/k$, hence a square unit cell of area d^2 .

The main results of this Letter are based on the numerical calculation of the energy bands and of the associated Bloch states for a two-level atom moving in the potential $V(\mathbf{r})$. This will be done without any assumption regarding the adiabatic following of the dark state. However, it is interesting to start with the case when this adiabatic following is assumed. The adiabatic following of the dark state $|\mathcal{D}(\mathbf{r})\rangle$ gives rise to a synthetic magnetic field (the Berry curvature) [18], whose flux across the unit cell is quantized by an integer N_ϕ times the magnetic flux quantum h . The integer N_ϕ corresponds to the number of times the dark state wraps the Bloch sphere associated with the pseudospin $|g_{\pm}\rangle$ when moving across the unit cell [3]. Here, one can check that $N_\phi = 1$, a nonzero value that characterizes optical flux lattices. For example, the north pole $|g_+\rangle$ is reached when $\mathbf{r} \rightarrow \mathbf{0}$ and the south pole $|g_-\rangle$ on the edges of the primitive unit cell $\mathbf{r} = \pm(d/2)\mathbf{e}_x$ or $\pm(d/2)\mathbf{e}_y$.

This nonzero flux is one of the requirements for an OFL. Another requirement formulated in [3] is that the eigenstates of the coupling $\hat{V}(\mathbf{r})$ should be nondegenerate at any point \mathbf{r} . This second requirement is clearly not fulfilled for the choice (2–3) since \hat{V} vanishes at the unit cell corner $\mathbf{r} \rightarrow d(\mathbf{e}_x + \mathbf{e}_y)/2$. In fact, this second requirement cannot be satisfied for a dark-state-based OFL [19]. As shown in Appendix B, the bright-state admixture remains small at any point of the unit cell, so that the local vanishing of \hat{V} does not prevent the occurrence of topological bands.

Note that the configuration studied here yields the maximum possible flux density from light—about one flux quantum per λ^2 , where λ is the wavelength of the beams forming the lattice. This contrasts with early proposals for artificial gauge fields using dark states [22,23] (see also Ref. [24] for a non-Abelian version). Those relied on configurations where E_{bright} remained nonzero everywhere and assumed adiabatic following of $|\mathcal{D}(\mathbf{r})\rangle$, producing much lower flux densities—about one flux quantum per w^2 or λw , with w the laser beam size.

Band structure—We show in Fig. 1(c) the band spectrum for a two-level atom for the Hamiltonian $\hat{p}^2/(2m) + \hat{V}(\mathbf{r})$ as a function of the light coupling amplitude V_0 . Here, energies are expressed in units of the cyclotron frequency $\omega_c = 2\pi\hbar/(md^2)$ expected for a Landau level with the same magnetic flux density as our OFL. For large couplings $V_0 \gg \hbar\omega_c$, we find a series of narrow energy bands at low energy, with an almost uniform spacing matching the cyclotron gap $\hbar\omega_c$. These bands essentially populate the dark state, with a very small residual light shift, shown in Fig. 1(d) for $V_0 = 100 \hbar\omega_c$. For this light coupling, the ground band exhibits a width $\simeq 0.10 \hbar\omega_c$ (about 8 times smaller than the gap to the first excited band), and the mean

light shift is as low as $\langle \hat{V} \rangle \simeq 3 \times 10^{-4} V_0$. The mixing between dark and bright states occurs at a higher energy, which can be estimated by computing the spectrum of an atom subjected to the lattice potential given by the bright-state energy $E_{\text{bright}}(\mathbf{r})$. In the large V_0 limit, one can neglect tunneling between the different wells of the bright-state lattice, and one expects a harmonic spectrum for that state of frequency $\Omega_{\text{bright}} = \sqrt{2\pi V_0 \omega_c / \hbar}$ [dashed red lines in Fig. 1(c)].

To characterize the topological nature of the bands, we compute their Chern number and find the value $C = 1$ for the first 19 bands at $V_0 = 100 \hbar \omega_c$, confirming the similarity with Landau levels. Furthermore, the Berry curvature of the ground band is relatively uniform, with an rms deviation equal to $\simeq 12\%$ of its mean value. We study in Supplemental Material [19] the quantum geometrical tensor, which can be used to characterize more precisely the analogy between the low-lying bands of our OFL and Landau levels.

A general comparison between OFL bands and ideal Chern bands is provided in [25], where it is shown that for the lattice described by Eqs. (2) and (3), the width of the ground band tends to 0 as $V_0 / \hbar \omega_c \rightarrow \infty$. Furthermore, drawing on concepts from moiré materials, the authors of [25] demonstrate that, for a fixed V_0 , this width can be reduced to extremely low values by introducing additional light beams, thereby reinforcing the proximity to the Landau level structure.

Connection with Landau level orbitals—We can extend the comparison between the OFL and Landau levels by inspecting individual Bloch states in real space. We first recall the Landau level formalism in a way that is well suited for comparison with flux lattices, i.e., a spatially periodic problem. We focus here on the ground band, the lowest Landau level (LLL). We assume a magnetic field $\mathbf{B} = B\mathbf{e}_z$ and adopt the Landau gauge $\mathbf{A} = -By\mathbf{e}_x$, for which the system is invariant upon x translations $t_x(d_x) = \exp(-d_x \partial_x)$ of arbitrary distance d_x . The resulting conservation of the canonical momentum p_x yields the LLL basis states $|\phi_{p_x}\rangle$, of wave function

$$\phi_{p_x}(x, y) = \frac{e^{ip_x x / \hbar}}{\pi^{1/4} \sqrt{\ell}} \exp\left[-\frac{(y - p_x \ell^2 / \hbar)^2}{2\ell^2}\right], \quad (4)$$

where ℓ is the magnetic length. The basis $|\phi_{p_x}\rangle$ is not convenient for comparison with OFL Bloch bands, which are indexed by a two-dimensional (2D) quasimomentum \mathbf{q} . In order to symmetrize the x and y directions, we use another symmetry of the system, namely the y -magnetic translation symmetry $t_y(d_y) = \exp[-d_y(\partial_y - ix/\ell^2)]$ [26]. Translation symmetries obey $t_x(d_x)t_y(d_y) = e^{-id_x d_y / \ell^2} t_y(d_y)t_x(d_x)$, hence a common basis of x and y translations can be found by choosing $d_x = d_y = d = \sqrt{2\pi}\ell$, such that the $d \times d$ unit cell is thread by a

single unit of flux quantum Φ_0 . Magnetic Bloch states $|\psi_{q_x, q_y}\rangle$ invariant under both $t_x(d)$ and $t_y(d)$ can be formed by discrete sum of momentum states $|\phi_{p_x}\rangle$, as

$$|\psi_{q_x, q_y}\rangle = \sqrt{d} \sum_{n=-\infty}^{\infty} e^{iq_y d n} |\phi_{q_x + nQ}\rangle \quad (5)$$

with $Q = 2\pi/d$. This expression corresponds to a discrete Fourier transform of the momentum states $|\phi_{q_x + nQ}\rangle$ whose density probabilities are shifted one to the other along y by integer multiples of d . The states $|\psi_{q_x, q_y}\rangle$, defined in the Brillouin zone $|q_x|, |q_y| < \pi/d$, form a basis of the LLL. We note that the wave functions match the LLL states defined on a torus of area $d \times d$ thread by magnetic fluxes $2\pi(\Phi_x, \Phi_y)/\Phi_0 = (q_x, q_y)d$, given by [27]

$$\psi_{q_x, q_y}(x, y) = \frac{e^{-y^2/(2\ell^2) - iq_x q_y d^2/(2\pi)}}{\pi^{1/4} \sqrt{\ell}} \Theta \left[\begin{matrix} q_x d \\ q_y d \end{matrix} \right] \left(\frac{x - iy}{d} \middle| i \right),$$

where $\Theta \left[\begin{matrix} a \\ b \end{matrix} \right] (z|\tau) = \sum_{n=-\infty}^{\infty} e^{i\pi n(n+a)\tau} e^{i2\pi n(n+a)(z+b)}$ is the Jacobi function [28].

Vortex structure—We can now compare the properties of magnetic Bloch states for the OFL and LLL. The state $\mathbf{q} = \mathbf{0}$ of the OFL exhibits, within each unit cell, a highly contrasted dip of the total density [see Fig. 2(a)], around which the velocity circulates [see Fig. 2(b)]. A similar behavior is found for different quasimomentum states, albeit with a different hole position. This behavior strongly resembles magnetic Bloch states of the LLL, which exhibit one quantized vortex point defect per unit cell, at which the density exactly cancels [see Fig. 2(c)] and around which the complex phase θ winds once by 2π . The velocity field, given by

$$\mathbf{v} = \frac{1}{m|\psi|^2} \text{Re} \left[\psi^* \left(\frac{\hbar}{i} \nabla - q\mathbf{A} \right) \psi \right] = \frac{\hbar}{m} \nabla \theta - \frac{q\mathbf{A}}{m}, \quad (6)$$

is shown in Fig. 2(d). We note that the circulation of velocity around each unit cell cancels due to the compensation between the singular contribution of the vortex and the rotational flow induced by the magnetic field. Each magnetic Bloch state thus exhibits nonzero local currents within the magnetic unit cell, but no global flow on larger length scales. The occurrence of vortices in OFL orbitals provides another illustration of the “ideal” Chern band character: like in the LLL, OFL orbitals can be fully characterized by the location of vortices [29].

Topological signatures with quantum gases—We now consider experimental signatures of the topological character of the low-energy bands of the OFL. We first consider a low-temperature Fermi gas in a harmonic trap $U(\mathbf{r})$ and subjected to the OFL. We show in Fig. 3(a) its total density variation, assuming a chemical potential μ_0 set in the gap separating the ground and first excited bands. In the local density approximation framework, we consider the system

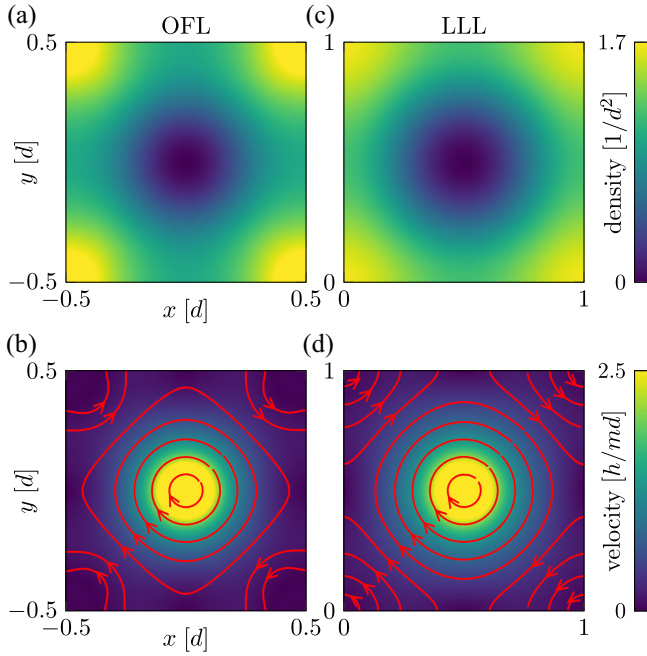


FIG. 2. Mean density (a),(c) and velocity (b),(d) distributions on a square unit cell $d \times d$ threaded by one unit of magnetic flux quantum. The left (respectively, right) panels correspond to the lowest band of the OFL for $V_0 = 100 \hbar\omega_c$ (respectively, the LLL). In (b),(d), the color encodes the norm of the velocity and the red arrows the velocity streamlines. Similar to the vortex singularity encountered in the LLL, the unit cell of the OFL exhibits a highly contrasted density dip around which the velocity circulate.

at position \mathbf{r} as locally homogeneous with a chemical potential $\mu(\mathbf{r}) = \mu_0 - U(\mathbf{r})$. In the central disk defined by $\mu(\mathbf{r})$ higher than the ground band energy, the Fermi sea forms a band insulator, with a uniform coarse-grained density [see Fig. 3(c)]. Outside this disk, the density drops to zero, and the system develops on the edge a chiral current consistent with the bulk-edge correspondence [see Fig. 3(b)] [30,31].

We also consider a Bose-Einstein condensate prepared in similar conditions, in the presence of weak repulsive interactions. The condensate wave function is computed by evolving the Gross-Pitaevskii equation in imaginary time using a split-step method. The *in situ* density profile, shown in Fig. 4(a), exhibits a square lattice of density holes corresponding to the regular array of quantized vortices expected for the quasimomentum $\mathbf{q} = \mathbf{0}$. We point out that, in our calculation, interactions are strong enough to significantly promote atoms to the first excited band (chemical potential $\mu \simeq 1.34 \hbar\omega_c$), which leads to a reduction of the vortex core size compared to the state $\mathbf{q} = \mathbf{0}$ of the ground band, shown in Fig. 2(a).

This vortex lattice shares similarities with those observed in rotating Bose gases [32–34], albeit arranged here on a square lattice imposed by the OFL geometry. In practice, vortices can be revealed as holes in the density profile [35].

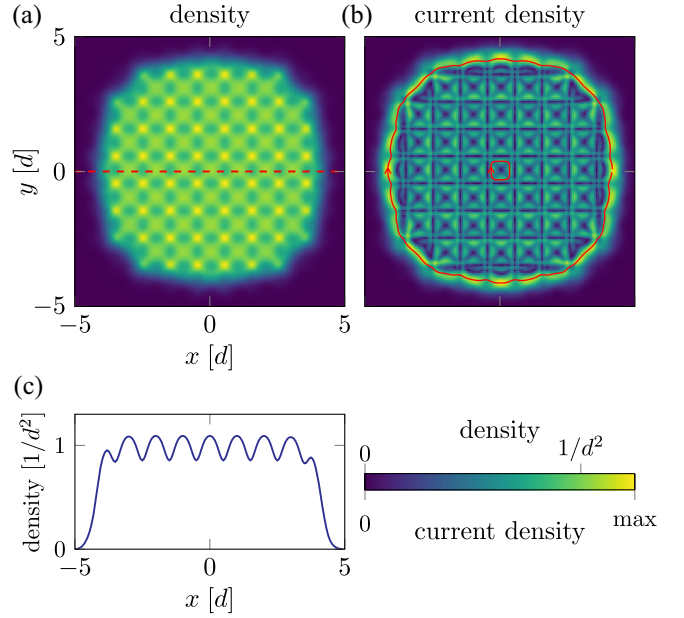


FIG. 3. *In situ* density (a) and current density (b) profile of a Fermi gas at zero temperature in a quasi-2D geometry subjected to the optical flux lattice with $V_0 = 100 \hbar\omega_c$ and to an isotropic harmonic trap (trapping frequency $\omega_{\text{trap}} = 0.04\omega_c$). We used a chemical potential $\mu_0 = 1.79 \hbar\omega_c$ located between the ground and first excited bands. A few streamlines are shown as red lines in (b). (c) Density profile along the line $y = 0$, showing incompressibility in the bulk.

One can also consider observing them after a time-of-flight (TOF) expansion in order to magnify them and reveal the associated quantized phase winding. In our OFL implementation, the velocity distribution cannot be used as such to reveal the vortex lattice, because it only expands over discrete Bragg peaks [see Fig. 4(b)] [36]. On the other hand, it was shown in [37] that any state of the lowest Landau level in the symmetric gauge expands in time of flight in a self-similar manner. In order to retrieve the vortex lattice configuration in time of flight, the condensate wave function should be multiplied by the factor $\exp(ixy/d^2)$ corresponding to the gauge transform from the Landau to symmetric gauge, which can be performed by applying a quadrupolar potential Qxy for a short duration prior to expansion. This is confirmed by calculating the velocity distribution of the condensate wave function multiplied by the complex phase factor [see Fig. 4(c)]. Note that triangular vortex arrays can also be achieved with dark-state lattices [19,25].

Implementation—Different optical setups can be envisioned for implementing the proposed OFL. Here we consider an atom that possesses a narrow transition between a ground level G with angular momentum J_G and an excited level E with angular momentum $J_E = J_G - 1$, as found in lanthanide species such as dysprosium or erbium. We extract a Λ system by considering the three magnetic states $|g_{-}\rangle \equiv |G, m = J_G - 1\rangle$, $|g_{+}\rangle \equiv |G, m = J_G\rangle$, and

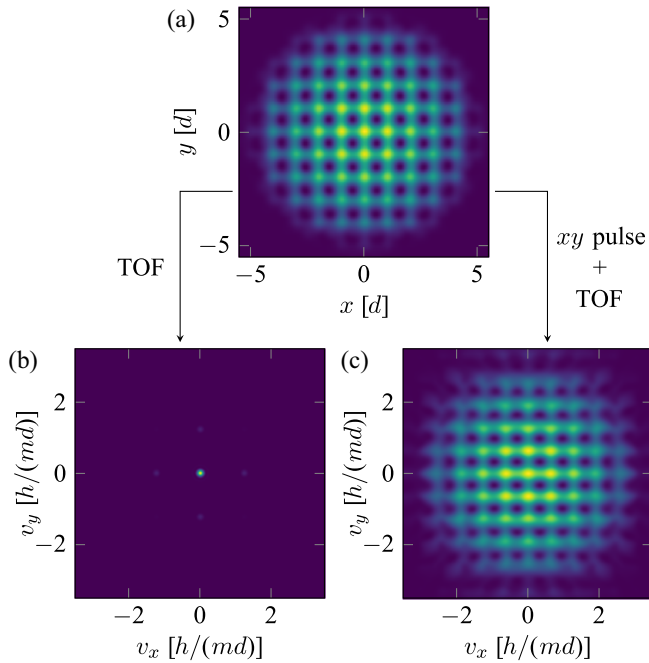


FIG. 4. (a) *In situ* density profile of a Bose-Einstein condensate in a quasi-2D geometry subjected to the optical flux lattice with $V_0 = 50 \hbar\omega_c$ and to an isotropic harmonic trap. We used a trapping frequency $\omega_{\text{trap}} = 0.06\omega_c$ and an interaction coupling constant g such that $gN/d^2 = 100 \hbar\omega_c$, where N is the number of atoms per spin state. (b) Velocity distribution obtained after a free expansion. The discrete Bragg peaks do not contain information on the vortex lattice structure prior expansion. (c) Velocity distribution obtained after a short pulse of quadrupolar potential Q_{xy} mapping the condensate wave function to the symmetric gauge. In that case, velocity distribution reveals the vortex lattice structure.

$|e\rangle \equiv |E, m = J_E\rangle$. The degeneracy between $|g_{\pm}\rangle$ is lifted using a static magnetic field oriented along the z axis, and the other states from the G and the E manifolds are put out of resonance using the light shift of an auxiliary off-resonant laser beam. All laser beams creating the OFL propagate in the x - y plane. Those driving the transition $|g_{-}\rangle \leftrightarrow |e\rangle$ are linearly polarized along z , as needed for a $\Delta m = 0$ transition. For the beams driving the transition $|g_{+}\rangle \leftrightarrow |e\rangle$, we choose an in-plane polarization that can be decomposed as left- and right-handed circular polarization (σ_{\pm}) along the z axis. Only the σ_{-} component is relevant for the considered transition. As an example, we discuss in Appendix C the practical implementation for Dy atoms and estimate the residual photon scattering rate ($\approx 1 \text{ s}^{-1}$) for realistic experimental parameters.

We also considered the effect of imperfect realization of the atom-laser coupling and the influence of a residual deviation from the Raman resonance transition on the band structure. As discussed in Supplemental Material [19], fine-tuning of these parameters can be used to flatten the ground band dispersion, and topological features are robust with respect to moderate uncontrolled imperfections.

In conclusion, we have proposed a generalization of optical flux lattices to a dark internal state of an atom. This scheme gives rise to continuum topological bands even in the presence of a strong optical lattice. Those bands can be described as ideal Chern bands to a very good approximation, so that they inherit the algebraic structure of Landau levels. As shown in [38], this property guarantees the existence of many-body ground states described by model wave functions, such as the bosonic Laughlin state at half filling. This, combined with the strong reduction of spontaneous emission in dark internal states, makes our proposal appealing for the simulation of quantum Hall effect in atomic gases.

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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End Matter

Appendix A: Dark state of a Λ transition—In this paragraph, we consider the Λ transition sketched in Fig. 1(a), adding the possibility of a nonzero Raman

detuning, with $\Delta_+ \equiv \omega_1 - \omega_+$ differing slightly from $\Delta_- \equiv \omega_2 - \omega_-$. In the dressed atom picture [16], denoting n_{\pm} the number of photons in each light wave,

the laser excitation couples the three states

$$|g_+, n_+ + 1, n_-\rangle \leftrightarrow |e, n_+, n_-\rangle \leftrightarrow |g_-, n_+, n_+ + 1\rangle \quad (\text{A1})$$

with the space-dependent Rabi frequencies $\kappa_{\pm}(\mathbf{r})$. The laser coupling reads in this three-dimensional space

$$\hat{W}(\mathbf{r}) = \hbar \begin{pmatrix} +\delta/2 & \kappa_+^* & 0 \\ \kappa_+ & -\Delta - i\Gamma/2 & \kappa_- \\ 0 & \kappa_-^* & -\delta/2 \end{pmatrix}, \quad (\text{A2})$$

where we have introduced the average detuning $\Delta = (\Delta_+ + \Delta_-)/2$ and the detuning from the Raman resonance $\delta = \Delta_+ - \Delta_-$. We also added an imaginary component $-i\Gamma/2$ to the energy of the excited state e to take into account its finite radiative lifetime Γ^{-1} . For $\delta = 0$, the state $|\mathcal{D}(\mathbf{r})\rangle \propto \kappa_-(\mathbf{r})|g_+\rangle - \kappa_+(\mathbf{r})|g_-\rangle$ is an eigenstate with energy 0 of the matrix \hat{W} .

In the limit $|\Delta| \gg |\kappa_{\pm}|, |\delta|, \Gamma$, we can eliminate the excited state adiabatically and obtain an effective Hamiltonian acting in the $\{|g_+\rangle, |g_-\rangle\}$ basis,

$$\hat{V}(\mathbf{r}) = \frac{1}{2}\hbar\delta\hat{\sigma}_z + \frac{\hbar}{\Delta} \begin{pmatrix} |\kappa_+|^2 & \kappa_+\kappa_- \\ \kappa_+\kappa_-^* & |\kappa_-|^2 \end{pmatrix}, \quad (\text{A3})$$

where $\hat{\sigma}_z$ is the Pauli matrix. We recover the light coupling formula (1) upon the identification $\kappa_{\pm} = \sqrt{\Delta/V_0}\hbar\alpha_{\pm}$. In the main text, we restrict to the case where the Raman condition is fulfilled ($\delta = 0$), which directly leads to the structure (1) for the atom-laser coupling. In Appendix C we evaluate the effect of the finite lifetime of the excited state by solving the eigenvalue problem for the full Hamiltonian $\hat{p}^2/2m + \hat{W}(\mathbf{r})$. The effect of a nonzero Raman detuning δ is addressed in Supplemental Material [19].

Appendix B: Admixture with the bright state—In this paragraph, we discuss the admixture of the quantum states of the ground band of the dark optical lattice with the bright subspace of the Λ transition.

In a first step, we consider the overlap $\mathcal{O}_{\text{bright}}(\mathbf{q})$ of a given Bloch state of quasimomentum \mathbf{q} with the bright subspace, plotted in Fig. 5(a). The bright-state admixture remains very small for all values of quasimomentum. We checked that it decays with the lattice depth as $\mathcal{O}_{\text{bright}}(\mathbf{q}) \propto \hbar\omega_c/V_0$, in agreement with the first correction to the Born-Oppenheimer approximation.

Remarkably, we find that the bright-state admixture cancels at the corner $\mathbf{q} = (\pi/d, \pi/d)$ of the first Brillouin zone. This behavior can be explained by the fact that the position-dependent dark state

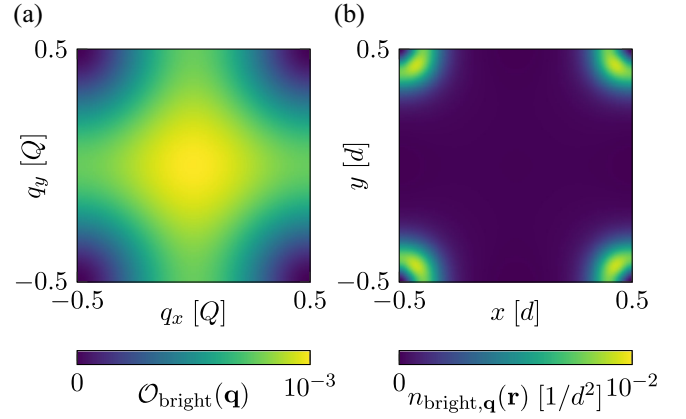


FIG. 5. (a) Overlap $\mathcal{O}_{\text{bright}}(\mathbf{q})$ of a given Bloch state of quasimomentum \mathbf{q} of the lowest band with the bright subspace, computed for $V_0 = 100 \hbar\omega_c$. (b) Density probability in the bright subspace $n_{\text{bright},\mathbf{q}}(\mathbf{r})$ for the quasimomentum $\mathbf{q} = 0$.

$$|\mathcal{D}(\mathbf{r})\rangle \propto \alpha_-(\mathbf{r})|g_+\rangle - \alpha_+(\mathbf{r})|g_-\rangle$$

$$\propto \left[e^{i\frac{k}{2}(x-y)} + e^{i\frac{k}{2}(-x+y)} + e^{i\frac{k}{2}(x+y)} + e^{i\frac{k}{2}(-x-y)} \right] |g_+\rangle + \left[e^{i\frac{k}{2}(x-y)} - e^{i\frac{k}{2}(-x+y)} + ie^{i\frac{k}{2}(x+y)} - ie^{i\frac{k}{2}(-x-y)} \right] |g_-\rangle$$

is an exact eigenstate of the Hamiltonian. Indeed, it is by construction the zero-energy eigenstate of the light-shift operator $\hat{V}(\mathbf{r})$ for all positions \mathbf{r} . Since it only involves plane waves of momentum $(k/2)(\pm\mathbf{e}_x \pm \mathbf{e}_y)$, it is also an exact eigenstate of the kinetic energy operator with energy E_r , where $E_r \equiv \hbar^2 k^2/(4m)$ is the recoil energy for this transition (recall that $k = k_1/\sqrt{2}$). Finally, it corresponds to a quasimomentum $\mathbf{q} = (\pi/d, \pi/d)$, hence it matches with the quasimomentum state of the lowest Bloch band at the corner of the first Brillouin zone.

In a second step, we study the locations in real space where the admixture of a Bloch state of quasimomentum $\mathbf{q} \neq (\pi/d, \pi/d)$ is maximal. For this, we compute numerically the position-resolved density probability for a given quasimomentum state \mathbf{q} of the lowest band to be found in the bright subspace,

$$n_{\text{bright},\mathbf{q}}(\mathbf{r}) = |\langle \psi_{\mathbf{q}}(\mathbf{r}) | \mathcal{B}(\mathbf{r}) \rangle|^2.$$

We find that this admixture predominantly occurs in the vicinity of the unit cell corner $d(\mathbf{e}_x + \mathbf{e}_y)/2$ [see Fig. 5(b)]. It can be explained by the fact that these corners are the locations where the Wannier functions of the lowest energy level for the bright state are maximal. Hence, for an atom moving in the LLL associated with the dark state, the probability to have a nonadiabatic transfer to these Wannier functions is maximal at the unit cell corners.

Appendix C: Implementation with dysprosium atoms—The dark optical lattice studied in the main text involves

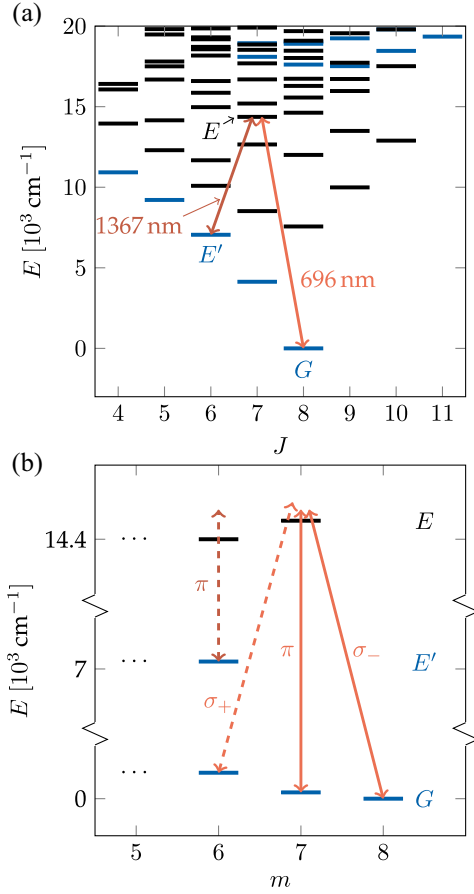


FIG. 6. (a) Level scheme of atomic dysprosium up to a $20\,000\text{ cm}^{-1}$ energy [39]. Even (odd) parity levels are shown in blue (black, respectively). We label the three energy levels G , E , and E' considered in Appendix C, together with the two optical transitions connecting them. (b) Scheme of the magnetic levels considered in Appendix C. The three levels $|G, m = 8\rangle$, $|G, m = 7\rangle$, and $|E, m = 7\rangle$ forming the Λ configuration are coupled using off-resonant light at 696 nm with polarization π and σ_- with respect to the applied quantization magnetic field. The dashed arrows represent additional light couplings used to lift the energy of the undesired states $|G, m = 6\rangle$ and $|E, m = 6\rangle$.

a triple of atomic states in a Λ configuration. Lanthanide atoms, which exhibit a rich electronic structure with isolated narrow optical transitions, are well suited for isolating such a triple of states. We take here the example of dysprosium atoms, whose electronic structure is shown in Fig. 6(a). For sake of simplicity, we consider bosonic atoms, of nuclear spin $I = 0$, but the scheme can be straightforwardly extended to fermionic isotopes.

The ground electronic level G exhibits an angular momentum $J_G = 8$. For the two ground states of the Λ configuration, we consider the two magnetic states $|g_- \rangle \equiv |G, m = J_G - 1\rangle$ and $|g_+ \rangle \equiv |G, m = J_G\rangle$. The excited

level is defined as the stretched state $|e \rangle \equiv |E, m = J_E\rangle$ of the excited level of energy $14\,368\text{ cm}^{-1}$ above the ground state, and angular momentum $J_E = 7$. These three states can be coupled using coherent light of wavelength $\lambda = 696\text{ nm}$ with π and σ_- polarization. As discussed in the main text, this light must be blue detuned with respect to the optical resonance, with a detuning Δ much larger than the light shift V_0 , the cyclotron frequency $\omega_c \simeq h \times 1.6\text{ kHz}$, and the excited-state inverse lifetime $\Gamma \simeq 94\text{ ms}^{-1}$.

The Landau level structure, with quasiflat energy bands, can be deformed due to the presence of additional magnetic levels, such as the ground-state level $|G, m = J_G - 2\rangle$ and the excited state $|E, m = J_E - 1\rangle$. Those states must be shifted in energy using additional laser beams in order to suppress their influence on the low-lying band structure.

More specifically, the state $|G, m = J_G - 2\rangle$ must be shifted to an energy $\Delta_1 \gg E_{\text{rec}}$, which can easily be done using a blue-detuned light beam at 696 nm with σ_+ polarization, without affecting the states $|g_{\pm}\rangle$ [40].

The state $|E, m = J_E - 1\rangle$ is off-resonantly coupled to the state $|g_- \rangle$ by the σ_- polarized light at 696 nm , leading to an additional light shift that can potentially deform the low-lying bands. This effect can be made negligible by shifting the energy of this state by a value $\Delta_2 < 0$, with $|\Delta_2|$ largely exceeding the detuning Δ . This can be achieved by off-resonant coupling this state to another electronic level $|E', m = J_{E'}\rangle$, of energy $7\,051\text{ cm}^{-1}$ above the ground state and angular momentum $J_{E'} = 6$. By using a light polarization π , the excited level $|e \rangle$ remains unaffected.

Finally, we estimate the photon scattering rate induced by the off-resonant population of excited levels. For this, we compute the band structure including the levels $|g_- \rangle$, $|g_+ \rangle$, $|e \rangle$, $|G, m = J_G - 2\rangle$, and $|E, m = J_E - 1\rangle$, including an imaginary part $i\Gamma/2$ in the energy of the excited states. We obtain a band structure with quasiflat energy bands for $V_0 = 100\hbar\omega_c$, a detuning $\Delta = 25V_0 \simeq 2\pi \times 4\text{ MHz}$, and light shifts of undesired states $\Delta_1 = 2\pi \times 100\text{ kHz}$ and $\Delta_2 = -100\Delta \simeq 2\pi \times 400\text{ MHz}$. For these parameters, we compute a mean scattering rate $\Gamma_{\text{sc}} \simeq 1\text{ s}^{-1}$ from the imaginary part of the ground band eigenstates. We note that, given the small value of the detuning Δ , the contribution to the light shift and scattering rate from the other transitions is negligible [41]. The low value of the scattering rate makes this protocol promising for the investigation of interacting many-body ground states in topological bands.

Equally promising results are obtained for the fermionic isotope 87 of strontium, with the lattice light tuned to the intercombination line $^1S_0 - ^3P_1$, more specifically using the states $|g_- \rangle = |G, F = 9/2, m = 7/2\rangle$, $|g_+ \rangle = |G, F = 9/2, m = 9/2\rangle$, and $|e \rangle = |E, F' = 7/2, m' = 7/2\rangle$.